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Structural Analysis of Network Connections of Koleje Małopolskie sp. z o.o. as a Significant Element of the Management of the Transport Company

Abstract

Public transport is one of the most important elements of the economy and social space development in which it is located. Issues related to its proper organisation are very often the responsibility of transport companies, which must operate and carry out their activities within the principles of the economy, that is, the principles of managing a service company. An important element of an efficient and effective functioning of a transport company is the quality of the provided services. An element significantly affecting the quality of the services offered is, inter alia, an appropriately organised network of connections offered by the carrier. This network in an efficient and interference-resistant manner must facilitate transport in a given area. In the article, using the graph theory and the simulation method, the structure of network connections of a railway carrier operating in the Małopolska region (Koleje Małopolskie sp. z o.o.) in Poland was analysed. The aim of the analysis was to obtain information about the current condition and parameters of the network offered by the carrier. The conducted analysis made it possible to assess the current state as well as within the conducted research and it proposed to modify the existing network of connections in order to improve its parameters. The carrier's network is used, inter alia, to co-create public transport in the region, and the structure of the connections network and its appropriate planning can have a significant impact on the functioning of the company.

Keywords: public transport, rail transport, transport networks, Koleje Małopolskie sp. z o.o., railway network
JEL Classification Codes: L92

Introduction

Sustainable transport is now one of the European Union's priorities in the area of its current activities. The current transport situation in Poland and Europe connected with a very large use of individual transport for the implementation of everyday displacements and related negative effects in the form of, among others, environmental pollution, noise, smog and the effects of road accidents (e.g. accidents and related injuries, death, etc.) cause the European Union, as part of its transport policy, to try to limit the share of car transport (including the individual one) to other modes of transport.

The above activities are particularly important in large cities or agglomerations such as Kraków. In Kraków, actions have been taken for many years to overcome the negative effects of transport (mainly individual) on the environment and cultural heritage (including monuments), which the city is famous for. These activities include: designing bus corridors, charges for parking the car in the city centre (which also aims to discourage the use of individual transport), separation of corridors for trams, etc. Obivously, these activities have the effect of reducing the share of individual transport in public transport, but these are not sufficient actions. In connection with the above, the city authorities and voivodships undertake further actions in the field of promoting public transport for individual transport.

In addition, for several years in Poland, based on global trends, measures have been taken to diversify the modes of transport that implement public transport (both in the agglomeration and the region), by including in the public transport organization the so-called urban or regional rail transport. This is to increase transport accessibility in Poland.

Activities of this type were also undertaken in the Małopolska region by appointing the railway carrier – Koleje Małopolskie sp. z o.o. Based on the existing railway infrastructure, this carrier has created a network of connections in which it provides transport services. The layout of this network and its structure are a very important element affecting the level and quality of services provided, therefore, it is important and recommended to analyse its structure using currently used methods of network analysis – this analysis is the main goal of the article. The analysis was carried out using the so-called graph theory. The article, which is an introductory article on the analysis of the transport network of the Koleje Małopolskie sp z o.o. carrier, focused on the analysis of the connection network developed by the carrier and also after that analysis it proposed modifications the implementation of which may contribute to the improvement of network parameters and thus, the performance of the company, such as increasing the number of travelers.

Koleje Małopolskie

Koleje Małopolskie is a company that was established by the Authorities of the Małopolskie Voivodship in December 2013 to provide public services in the field of public transport by ensuring the effective organization and operation of passenger rail traffic in the Małopolskie Voivodship [Koleje Małopolskie, on-line, 28/03/2018]. In order to carry out its activities in June 2014, the company obtained a license to perform rail transport of persons No. WPO / 236/2014 [Koleje Małopolskie, on-line, 28/03/2018], in August 2014 obtained a European security certificate and in October 2014, a national safety certificate [Koleje Małopolskie, on-line, 28/03/2018].

As part of its activities, Koleje Małopolskie is tasked with providing transport services in the Małopolskie voivodship, but what is important, these services must also be provided in accordance with the principles of the free market economy. This situation causes the company, while undertaking its activities, to be forced to function as any enterprise whose primary goal is to develop and improve the quality of services provided, but also to obtain an economic benefit from the services provided. Thanks to these activities, the company will be able to provide services at a competitive level in relation to other carriers (operating, for example, in other branches, such as cars) and increase its market share.

Currently, the company has had only a few years of activity, but in this period has made its intensive development and at the moment already has a significant number of transported passengers and began to increase its transport performance. The detailed data on the number of transported passengers and transport performance is presented in Table 1 and graphically in Figures 1 and 2.

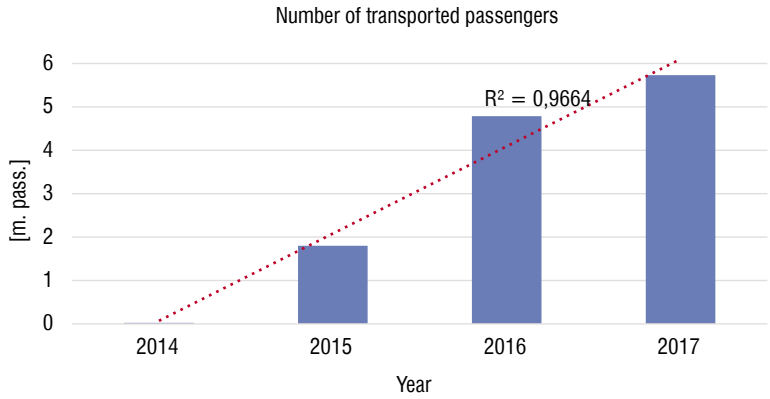
Table 1. The number of transported passengers and transport performance made by Koleje Małopolskie in years 2014–2017

	Year			
	2014	2015	2016	2017
Number of transported passengers [m]	0.026136	1.801765	4.788024	5.730428
Transport performance [m pass. km]	0.517178	28.399650	87.086964	168.350904

Source: Statistical data received electronically from the Office of Rail Transport on January 16, 2018.

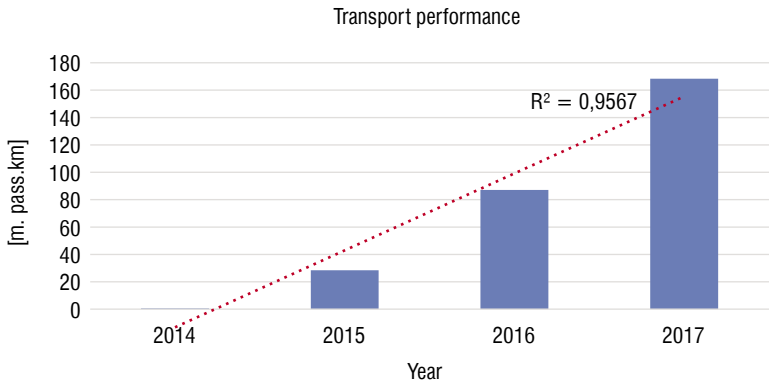
As shown in Table 1 and in Figures 1 and 2, Koleje Małopolskie since the beginning of its activity has shown an almost linear increase in the number of passengers and transport performance. This proves that the carrier is very well received by the passengers, which is certainly influenced by a lot of factors (including the effect of a new "player" on the market, greater flexibility of travel by increasing the choice of transport modes, the modern fleet used by the carrier, etc.).

Figure 1. The number of passengers transported by Koleje Małopolskie in the years 2014–2017



Source: the author's own elaboration based on Table 1.

Figure 2. The transport performance made by Koleje Małopolskie in the years 2014–2017



Source: the author's own elaboration based on Table 1.

However, in order to maintain a very good trend of increasing the number of passengers carried and the share in the local transport services market, further measures are necessary. One of such activities may be activities related to matching the offered network of connections with the needs and requirements of travelers as well as technical capabilities of the existing railway infrastructure managed by PKP PLK S.A., in whose infrastructure services are provided by the analysed carrier. On the absis of the above-mentioned railway network and in the voivodship, other rail carriers also provide their services, and what is more important, their services are also provided by carriers from other modes of transport. In connection with the above, the network of connections offered by the carrier should enable it to provide services both at a high level of reliability as well as at a high organisational level. For this purpose, the carrier's network should be analysed for some possible modifications to improve it. Undoubtedly, the structure of the connection network is not the only element that affects the quality and level of services provided. Many factors influence it, including nature of the area around

the given node (whether the existing facilities there may be generators of increased passenger traffic, e.g. hospitals, schools, universities, etc.), what is the capacity between the given node and neighboring ones, what is the travel speed as well as what is the technical level and layout of competitive transport branches in the local area. These are very important factors affecting the traffic flows generated and the choice of means of transport by the traveler, however, taking into account all these factors is a difficult and complicated issue, and sometimes even impossible to tackle due to, for example, a lack of access to the required reliable technical and statistical data (e.g. due to the so-called "business secret"). Taking into account the above-mentioned factors, the article focuses on the analysis of the carrier's transport network of connections, which is presented in Figure 3.

Figure 3 The carrier's current network of connections

Source: the author's own elaboration based on the railway infrastructure map in Poland and information placed on the website of the Małopolska Railways on April 5, 2018.

As shown in Figure 3, the connection network of Koleje Małopolskie was divided into 4 lines (depicted by 4 connection colours: blue, violet, orange and yellow).

The analysis of the above network will provide information about its individual parameters and enable to obtain information which nodes (stations) play a key role in the network and what kind of possible changes or modifications can be made to the network in order to improve it for better performance, which in the future may be translated into a further increase in the number of passengers travelling by this carrier.

The carrier's network of connections to obtain the above information may be analysed, among others using the graph theory.

Graph theory in the analysis of transport networks

The graph theory is used to analyse various types of networks, mainly social networks [Amaral L. et al., 2000, Newman M. et al., 2002, Arenas A. et al., 2003], but it is also used to analyse neural networks [Sporns O., 2002; Stam C. et al., 2007; Bullmore E. et al., 2009], and biological ones [Rual JF. Et al., 2005]. The analysis possibilities using the graph theory of the previously mentioned networks are also presented in [Newman M., 2010]. The graph theory is also used to analyse computer networks [Valverde S. et al., 2003] and transport networks [Newman M., 2010; Wilkinson S. et al., 2012; Li H. et al., 2014; Ouyang M. and others, 2015; Tarapata Z., 2015; Dunn, Wilkinson 2017].

The information on the role of individual nodes in the network obtained from the analysis allow initially estimating which of the points are particularly sensitive to potential threats or attacks [Newman M., 2010, Tarapata Z., 2015]. Most of the measures and calculations used also allow obtaining information which of the nodes of the network plays the main role or are a specific centre of the analysed network. According to the information contained in the literature [Tarapata Z., 2015], each network can be described as a set of nodes and links between them:

$$G = \langle V, E \rangle \tag{1}$$

where:
V – set of nodes;
E – set of connections between nodes.

Obviously, for every analysed network there is a dependence that:

$$|V| = N, \ |E| = M \tag{2}$$

where:
N – number of nodes in the network;
M – number of edges (connections) in the network.

The most frequently used indicators include those described in the literature [for example: Newman M., 2010, Tarapata Z., 2015], are:

1) Normalized degree dci i-th network node:

$$dc_i = \frac{k_i}{N-1} \tag{3}$$

where:
k_i – the degree of the i-th node in the network (the number of network node connections with other nodes);
N – number of nodes in the network.

The greater the value of the dc_i index for the i-th node, the node is more important in the network or closer to its centre.

2) Eccentricity ec_i *i*-th network node.

$$ec_i = \max_{j \in V} d_{ij} \tag{4}$$

where:

d_{ij} – the number of links between nodes that occurs on the shortest path between node *i* and *j*.

The lower the value of the ec_i index for the *i*-th node, the node is more important in the network or closer to its centre.

3) Radius rci i-th network node:

$$rc_i = \frac{1}{\max_{j \in V} d_{ij}} = \frac{1}{ec_i} \tag{5}$$

where:

d_{ij} – the number of links between nodes that occurs on the shortest path between node *i* and *j;*

ec_i – Eccentricity for the *i*-th network node.

The greater the value of the rc_i index for the *i*-th node, the node is more important in the network or closer to its centre.

4) Closeness cc_i:

$$cc_i = \frac{N-1}{\sum_{j \in V} d_{ij}} \tag{6}$$

where:

N – number of nodes in the network;

d_{ij} – the number of links between nodes that occurs on the shortest path between node *i* and *j*.

5) Beetweeness bc_i *i*-th network node:

$$bc_i = \sum_{l \in V} \sum_{k \neq l \in V} \frac{p_{l,i,k}}{p_{l,k}} \tag{7}$$

where:

$p_{l,i,k}$- number of connections with the shortest number of constraints between nodes *l* and *k* (containing node i);

$p_{l,k}$ – number of connections with the shortest number of constraints between nodes *l* and *k* (not including node i).

The greater the value of the bc_i index for the *i*-th node, the node is more important in the network or closer to its centre.

6) Clusterization gc_i i-th network node:

$$gc_i = \frac{2E_i}{k_i(k_i - 1)},\ k_i > 1 \tag{8}$$

where:

E_i – the number of bonds between the nodes that are closest to (neighbors) of the i-th node;

k_i – degree of the i-th node in the network (number of network node connections with other nodes).

The greater the value of the gc_i index for the i-th node, the node is more important in the network or closer to its centre.

Formulas (2)–(8) describe the parameters of individual network nodes, but in addition to them, coefficients are often used to determine the parameters of the entire analysed network. These are [Newman M., 2010, Tarapata Z., 2015]:

7) Average shortest paths length L:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j \in V} d_{ij} \tag{9}$$

where:

N – number of nodes in the network;

d_{ij} – the number of links between nodes that occurs on the shortest path between node i and j.

The shorter the average shortest paths length is, the better the analysed network is.

8) Clusterization coefficent C:

$$C = \frac{1}{N} \sum_{i \in V} gc_i \tag{10}$$

where:

N – number of nodes in the network;

gc_i – clusterization.

The higher the Clusterization coefficent value, the better the analysed network is.

9) Diameter D:

$$D = \max_{i \in V} ec_i \tag{11}$$

where:

ec_i – Eccentricity for the i-th network node.

The smaller the network diameter, the better the network.

10) Radius of a network R:

$$R = \min_{i \in V} ec_i \tag{12}$$

where:

ec_i – Eccentricity for the i-th network node.

The smaller the Radius of a network, the better the network.

11) Average nodes degree $\overline{k}$:

$$\overline{k} = \frac{1}{N}\sum_{i \in V} k_i \tag{13}$$

where:

N – number of nodes in the network;

k_i – degree of the i-th node in the network (number of network node connections with other nodes).

The calculation of the values of the above indicators makes it possible to assess the dependencies between individual network nodes as well as to determine what type of network we are dealing with and which network or which nodes in the network play an important role. In networks, there are practically no situations in which all network nodes have the same degree of "importance".

Each network has key nodes that are more than others responsible for the proper functioning of the entire network or are important in it for providing services in a key area. Determining these nodes and their locations allows drawing conclusions about the current state of the network and its resistance to possible interference. This information also allows entering or proposing improvements to the analysed network.

Analysis of the Koleje Małopolskie network

The network of connections of Koleje Małopolskie was analysed using the graph theory. The Freeware Gephi software was used to perform the calculation and visualization of the network.

For the connection network shown in Figure 3, the parameters of individual nodes (stations) were calculated and presented in Table 2.

Table 2. Parameters of individual network nodes (stations) for the analysed network of connections of Koleje Małopolskie

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Kraków Lotnisko	0.00952381	65	0.015384615	0.033903778	0	0	0.007164535
Kraków Zakliki	0.019047619	63	0.015873016	0.036319613	0.037728938	0	0.017865051
Kraków Olszanica	0.019047619	64	0.015625	0.035081858	0.019047619	0	0.012933559
Kraków Młynówka	0.019047619	62	0.016129032	0.037620924	0.056043956	0	0.035020425
Kraków Łobzów	0.019047619	61	0.016393443	0.038989974	0.073992674	0	0.137585316
Kraków Główny	0.057142857	60	0.016666667	0.040431267	0.35	0	0.736025324
Kraków Płaszów	0.057142857	58	0.017241379	0.042151746	0.412087912	0	0.899403844

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Kraków Prokocim	0.057142857	57	0.01754386	0.042909685	0.42014652	0	0.742916948
Kraków Bieżanów	0.057142857	56	0.017857143	0.043659044	0.478388278	0	0.518118741
Kraków Bieżanów Drożdżownia	0.019047619	57	0.01754386	0.04194966	0.056043956	0	0.104727468
Wieliczka Bogucice	0.019047619	58	0.017241379	0.040338071	0.037728938	0	0.029752948
Wieliczka Park	0.019047619	59	0.016949153	0.038817006	0.019047619	0	0.014729164
Wieliczka Rynek Kopalnia	0.00952381	60	0.016666667	0.03737985	0	0	0.007395471
Kraków Zabłocie	0.076190476	59	0.016949153	0.041387466	0.428937729	0	1
Skawina	0.00952381	65	0.015384615	0.033535612	0	0	0.007134922
Kraków Sidzina	0.019047619	64	0.015625	0.03468781	0.019047619	0	0.012656507
Kraków Swoszowice	0.019047619	63	0.015873016	0.035897436	0.037728938	0	0.0156532
Kraków Sanktuarium	0.019047619	62	0.016129032	0.037168142	0.056043956	0	0.019669449
Kraków Łagiewniki	0.019047619	61	0.016393443	0.03850385	0.073992674	0	0.042262614
Kraków Podgórze	0.019047619	60	0.016666667	0.03990878	0.091575092	0	0.182326965
Kraków Batowice	0.019047619	61	0.016393443	0.039340577	0.260805861	0	0.13761766
Zastów	0.019047619	62	0.016129032	0.038279256	0.247252747	0	0.035227552
Baranówka	0.019047619	63	0.015873016	0.037247251	0.233333333	0	0.018872085
Łuczyce	0.019047619	64	0.015625	0.036244391	0.219047619	0	0.016535259
Goszcza	0.019047619	65	0.015384615	0.035270406	0.204395604	0	0.016244752
Niedźwiedź	0.019047619	66	0.015151515	0.034324943	0.189377289	0	0.016213672
Słomniki Miasto	0.019047619	67	0.014925373	0.033407572	0.173992674	0	0.016210811
Słomniki	0.019047619	68	0.014705882	0.032517807	0.158241758	0	0.016210583
Smroków	0.019047619	69	0.014492754	0.03165511	0.142124542	0	0.016210564
Szczepanowice	0.019047619	70	0.014285714	0.030818902	0.125641026	0	0.016210529
Kamieńczyce	0.019047619	71	0.014084507	0.030008574	0.108791209	0	0.016210138
Miechów	0.019047619	72	0.013888889	0.02922349	0.091575092	0	0.016206433
Dziadówki	0.019047619	73	0.01369863	0.028462998	0.073992674	0	0.016177795
Tunel	0.019047619	74	0.013513514	0.027726433	0.056043956	0	0.016003402
Kozłów	0.019047619	75	0.013333333	0.027013121	0.037728938	0	0.015203531
Klimontów	0.019047619	76	0.013157895	0.026322387	0.019047619	0	0.012608882
Sędziszów	0.00952381	77	0.012987013	0.025653555	0	0	0.007130594
Kokotów	0.038095238	55	0.018181818	0.044247788	0.460805861	0	0.270312021
Węgrzyce Wielkie	0.038095238	54	0.018518519	0.044814341	0.466300366	0	0.1809432
Podłęże	0.038095238	53	0.018867925	0.045356371	0.471428571	0	0.151554099
Staniątki	0.038095238	52	0.019230769	0.04587156	0.476190476	0	0.143026726
Szarów	0.038095238	51	0.019607843	0.046357616	0.480586081	0	0.140887727
Kłaj	0.038095238	50	0.02	0.046812305	0.484615385	0	0.140427242
Stanisławice	0.038095238	49	0.020408163	0.047233468	0.488278388	0	0.140342028
Cikowice	0.038095238	48	0.020833333	0.047619048	0.491575092	0	0.140328287
Bochnia	0.038095238	47	0.021276596	0.047967108	0.494505495	0	0.140325494
Rzezawa	0.038095238	46	0.02173913	0.048275862	0.497069597	0	0.140318981

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Jasień Brzeski	0.038095238	45	0.022222222	0.048543689	0.499267399	0	0.140280827
Brzesko Okocim	0.038095238	44	0.022727273	0.048769159	0.501098901	0	0.140083543
Sterkowiec	0.038095238	43	0.023255814	0.048951049	0.502564103	0	0.139226646
Biadoliny	0.038095238	42	0.023809524	0.049088359	0.503663004	0	0.136154115
Bogumiłowice	0.038095238	41	0.024390244	0.049180328	0.504395604	0	0.127220392
Tarnów Mościce	0.038095238	40	0.025	0.049226442	0.504761905	0	0.106507052
Tarnów	0.028571429	39	0.025641026	0.049226442	0.504761905	0	0.068657598
Kłokowa	0.019047619	39	0.025641026	0.049180328	0.504395604	0	0.028652789
Łowczówek Pleśna	0.019047619	40	0.025	0.049088359	0.503663004	0	0.018706376
Łowczów	0.019047619	41	0.024390244	0.048951049	0.502564103	0	0.016624195
Tuchów	0.019047619	42	0.023809524	0.048769159	0.501098901	0	0.016267124
Lubaszowa	0.019047619	43	0.023255814	0.048543689	0.499267399	0	0.016217005
Siedliska k, Tuchowa	0.019047619	44	0.022727273	0.048275862	0.497069597	0	0.016211187
Chojnik	0.019047619	45	0.022222222	0.047967108	0.494505495	0	0.016210654
Gromnik	0.019047619	46	0.02173913	0.047619048	0.491575092	0	0.016211002
Bogoniowice Ciężkowice	0.019047619	47	0.021276596	0.047233468	0.488278388	0	0.016214774
Pławna	0.019047619	48	0.020833333	0.046812305	0.484615385	0	0.016244203
Bobowa	0.019047619	49	0.020408163	0.046357616	0.480586081	0	0.016426145
Bobowa Miasto	0.019047619	50	0.02	0.04587156	0.476190476	0	0.017284875
Jankowa	0.019047619	51	0.019607843	0.045356371	0.471428571	0	0.020243408
Wilczyska	0.028571429	52	0.019230769	0.044814341	0.525824176	0	0.027439155
Polna Szalowa	0.019047619	53	0.018867925	0.043334709	0.204395604	0	0.020243408
Wola Łużańska	0.019047619	54	0.018518519	0.041916168	0.189377289	0	0.017284875
Moszczenica	0.019047619	55	0.018181818	0.040556199	0.173992674	0	0.016426145
Małopolska	0.019047619	56	0.017857143	0.039252336	0.158241758	0	0.016244202
Gorlice	0.019047619	57	0.01754386	0.038002172	0.142124542	0	0.016214771
Zagórzany	0.019047619	58	0.017241379	0.036803365	0.125641026	0	0.016210962
Libusza	0.019047619	59	0.016949153	0.03565365	0.108791209	0	0.016210176
Biecz	0.019047619	60	0.016666667	0.034550839	0.091575092	0	0.016206436
Siepietnica	0.019047619	61	0.016393443	0.033492823	0.073992674	0	0.016177795
Skołoszyn	0.019047619	62	0.016129032	0.032477575	0.056043956	0	0.016003402
Przysieki	0.019047619	63	0.015873016	0.03150315	0.037728938	0	0.015203531
Jasło Niegłowice	0.019047619	64	0.015625	0.030567686	0.019047619	0	0.012608882
Jasło	0.00952381	65	0.015384615	0.029669398	0	0	0.007130594
Stróże	0.019047619	53	0.018867925	0.043768237	0.356043956	0	0.020243408
Grybów	0.019047619	54	0.018518519	0.042735043	0.345421245	0	0.017284875
Ptaszkowa	0.019047619	55	0.018181818	0.041716329	0.334432234	0	0.016426145
Mszalnica	0.019047619	56	0.017857143	0.040713455	0.323076923	0	0.016244203
Kamionka Wielka	0.019047619	57	0.01754386	0.039727582	0.311355311	0	0.016214774
Nowy Sącz Jamnica	0.019047619	58	0.017241379	0.03875969	0.299267399	0	0.016210999

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Nowy Sącz	0.019047619	59	0.016949153	0.037810587	0.286813187	0	0.016210603
Nowy Sącz Biegonice	0.019047619	60	0.016666667	0.036880927	0.273992674	0	0.016210569
Stary Sącz	0.019047619	61	0.016393443	0.035971223	0.260805861	0	0.016210566
Barcice	0.019047619	62	0.016129032	0.035081858	0.247252747	0	0.016210566
Rytro	0.019047619	63	0.015873016	0.034213099	0.233333333	0	0.016210566
Młodów	0.019047619	64	0.015625	0.03336511	0.219047619	0	0.016210566
Piwniczna Zdrój	0.019047619	65	0.015384615	0.032537961	0.204395604	0	0.016210566
Piwniczna	0.019047619	66	0.015151515	0.031731641	0.189377289	0	0.016210566
Łomnica Zdrój	0.019047619	67	0.014925373	0.030946065	0.173992674	0	0.016210566
Wierchomla Wielka	0.019047619	68	0.014705882	0.030181087	0.158241758	0	0.016210566
Zubrzyk	0.019047619	69	0.014492754	0.029436501	0.142124542	0	0.016210563
Żegiestów	0.019047619	70	0.014285714	0.028712059	0.125641026	0	0.016210529
Żegiestów Zdrój	0.019047619	71	0.014084507	0.028007469	0.108791209	0	0.016210138
Andrzejówka	0.019047619	72	0.013888889	0.027322404	0.091575092	0	0.016206433
Milik	0.019047619	73	0.01369863	0.026656512	0.073992674	0	0.016177795
Muszyna	0.019047619	74	0.013513514	0.026009413	0.056043956	0	0.016003402
Muszyna Zdrój	0.019047619	75	0.013333333	0.025380711	0.037728938	0	0.015203531
Powroźnik	0.019047619	76	0.013157895	0.024769993	0.019047619	0	0.012608882
Krynica Zdrój	0.00952381	77	0.012987013	0.024176836	0	0	0.007130594

Source: Own elaboration using Gephi software.

The figures show the distribution of sample parameters of individual network nodes. The larger the node size and the darker the colour, the larger the parameter value for the node is.

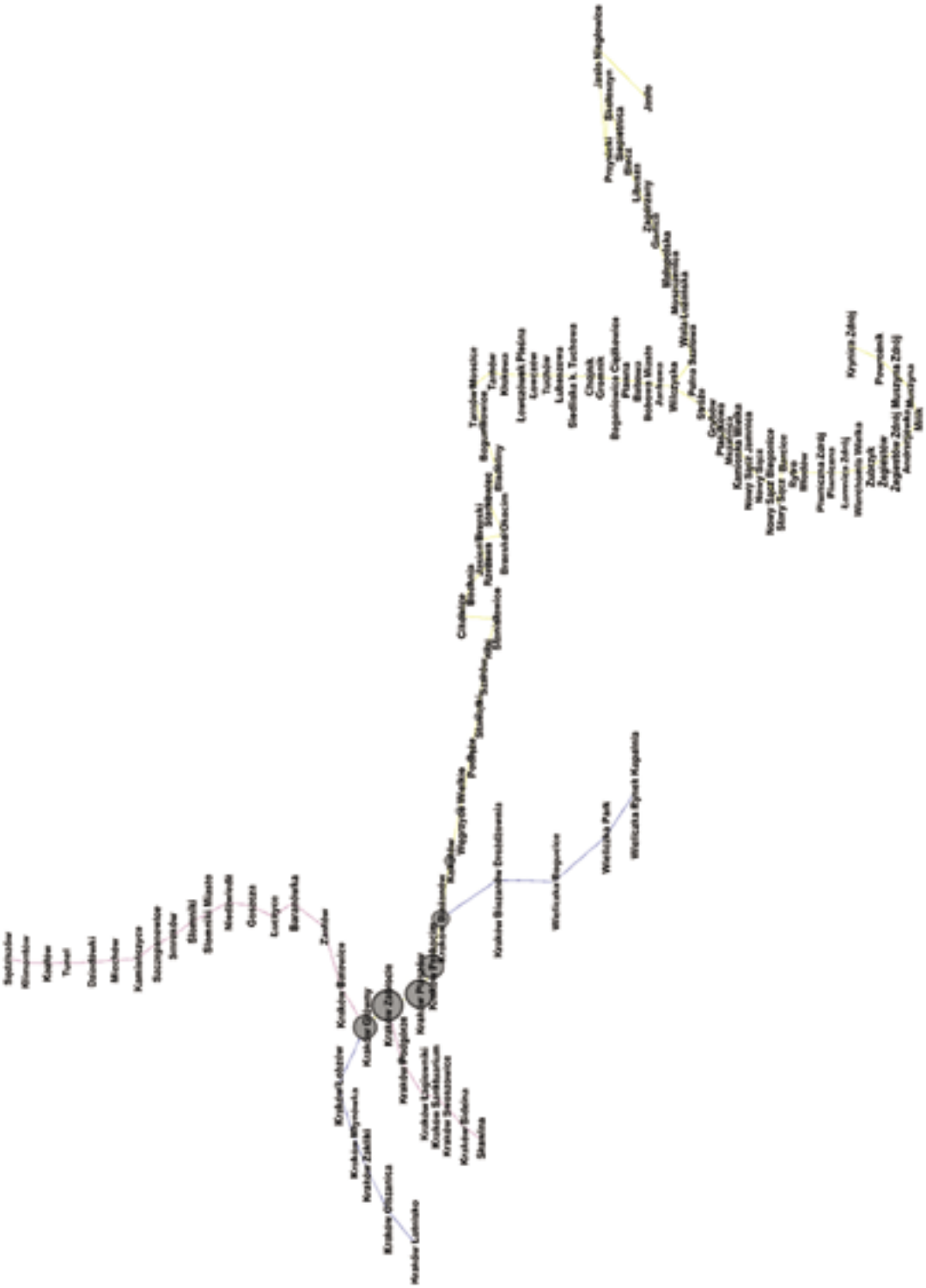
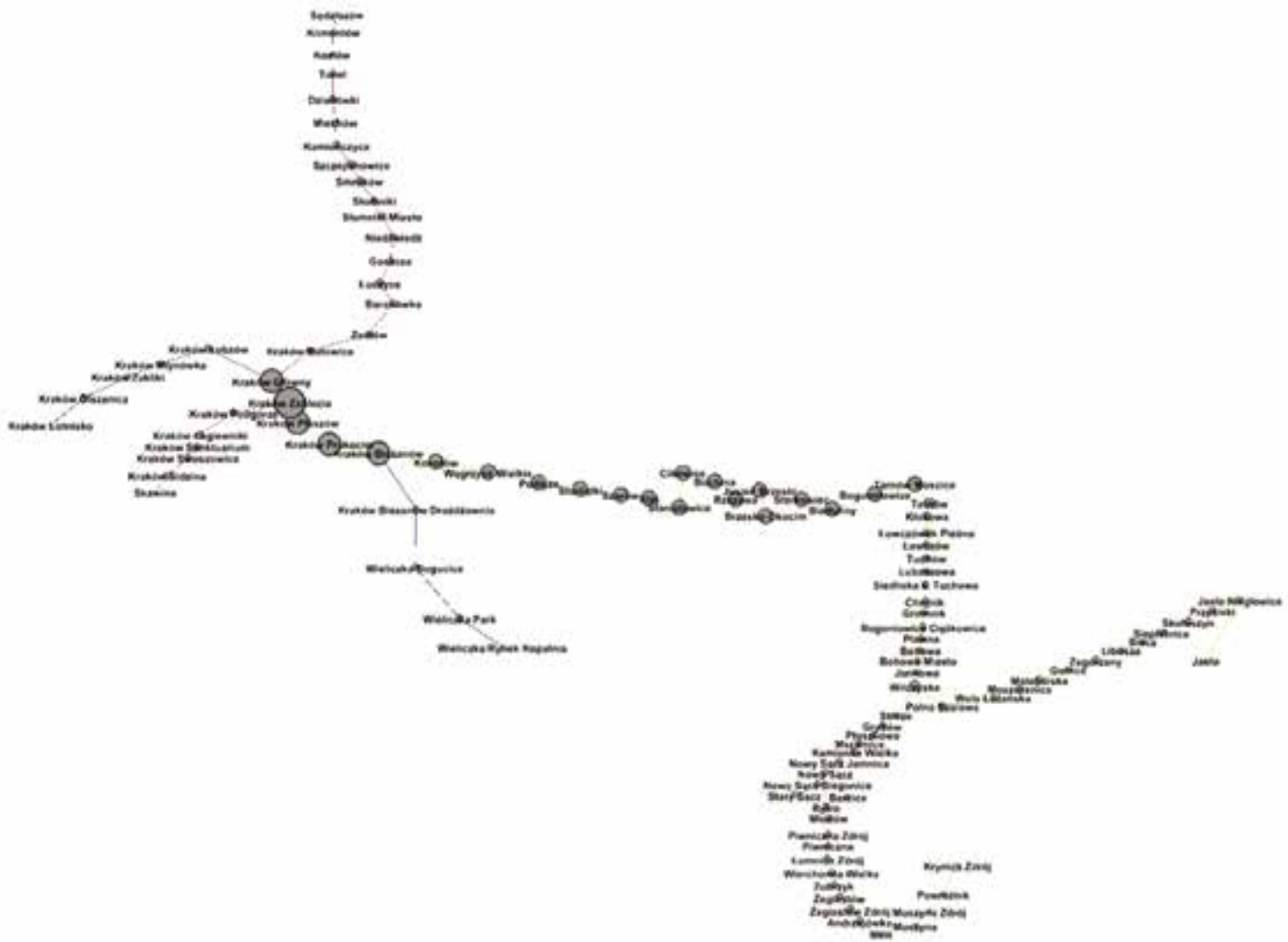


Figure 4. Eigencentrality for the analysed network

Source: the author's own study using the Gephi software.

Figure 5. Degree for the analysed network



Source: the author's own study using the Gephi software.

For the analysed network, its parameters as a whole were also calculated and presented in Table 3.

Table 3. Parameters of the analyzed network as a whole

Average Clustering Coefficent	Average path length	Diameter	Radius of a network	Average nodes degree
0	27.01437556	77	39	2.471698113

Source: the author's own elaboration based on table 2.

As presented in tables 2 and 3 and in Figures 4 and 5, the Kraków Główny, Kraków Zabłocie and Kraków Płaszów stations are the main and most important nodes in the analysed network.

Modification of the connection network of Koleje Małopolskie

Next, it was proposed to modify the network to improve its parameters. As part of the analyses, 2 modifications were proposed:

a) modification 1: added connections between Kraków Batowice and Węgrzyce Wielkie nodes, intermediate stations between them were also added (Dłubnia, Kraków Sambud, Kraków Lubocza, Kraków Nowa Huta, Kraków Nowa Huta Północ, Kraków Kościelniki, Przylasek Rusiecki, Podgrabie Wisła, Podgrabie Rudzice);
b) modification 2: connections between Kraków Łobzów and Kraków Zabłocie nodes have been added (with intermediate stations Kraków Olsza and Kraków Dąbie);

The proposed modifications are feasible because the railway infrastructure currently has a proper infrastructure which is managed by PKP PLK S.A.

Table 4 presents the results of the calculations for the connection network with the introduced modification 1.

Table 4. The parameters of individual network nodes (stations) for the analysed network connections of Koleje Małopolskie after the 1 modification.

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Kraków Lotnisko	0.008695652	65	0.01538462	0.035870243	0	0	0.00713439
Kraków Zakliki	0.017391304	63	0.01587302	0.038590604	0.034477498	0	0.017808284
Kraków Olszanica	0.017391304	64	0.015625	0.037192755	0.017391304	0	0.012880837
Kraków Młynówka	0.017391304	62	0.01612903	0.040069686	0.051258581	0	0.035043374
Kraków Łobzów	0.017391304	61	0.01639344	0.041636495	0.067734554	0	0.138132514
Kraków Główny	0.052173913	60	0.01666667	0.043298193	0.336668748	0	0.739029869
Kraków Płaszów	0.052173913	58	0.01724138	0.044992175	0.380488144	0	0.897166719
Kraków Prokocim	0.052173913	57	0.01754386	0.045780255	0.387200577	0	0.742016395
Kraków Bieżanów	0.052173913	56	0.01785714	0.046596434	0.439592426	0	0.521818157

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Kraków Bieżanów Drożdżownia	0.017391304	57	0.01754386	0.044642857	0.051258581	0	0.105111128
Wieliczka Bogucice	0.017391304	58	0.01724138	0.042814594	0.034477498	0	0.029711683
Wieliczka Park	0.017391304	59	0.01694915	0.041100786	0.017391304	0	0.014671765
Wieliczka Rynek Kopalnia	0.008695652	60	0.01666667	0.039491758	0	0	0.007363903
Kraków Zabłocie	0.069565217	59	0.01694915	0.044230769	0.400244055	0	1
Skawina	0.008695652	65	0.01538462	0.035341119	0	0	0.007104748
Kraków Sidzina	0.017391304	64	0.015625	0.036624204	0.017391304	0	0.012602706
Kraków Swoszowice	0.017391304	63	0.01587302	0.037978864	0.034477498	0	0.015582509
Kraków Sanktuarium	0.017391304	62	0.01612903	0.039410555	0.051258581	0	0.019575033
Kraków Łagiewniki	0.017391304	61	0.01639344	0.040925267	0.067734554	0	0.042095676
Kraków Podgórze	0.017391304	60	0.01666667	0.042529586	0.083905416	0	0.18201309
Kraków Batowice	0.026086957	61	0.01639344	0.042248347	0.280383945	0	0.150939079
Zastów	0.017391304	62	0.01612903	0.040983607	0.228832952	0	0.039367297
Baranówka	0.017391304	63	0.01587302	0.039764869	0.215713196	0	0.019875052
Łuczyce	0.017391304	64	0.015625	0.038590604	0.20228833	0	0.016674272
Goszcza	0.017391304	65	0.01538462	0.037459283	0.188558352	0	0.016203739
Niedźwiedź	0.017391304	66	0.01515152	0.036369386	0.174523265	0	0.016143731
Słomniki Miasto	0.017391304	67	0.01492537	0.03531941	0.160183066	0	0.016137172
Słomniki	0.017391304	68	0.01470588	0.034307876	0.145537757	0	0.016136558
Smroków	0.017391304	69	0.01449275	0.033333333	0.130587338	0	0.016136506
Szczepanowice	0.017391304	70	0.01428571	0.032394366	0.115331808	0	0.016136468
Kamieńczyce	0.017391304	71	0.01408451	0.031489595	0.099771167	0	0.016136084
Miechów	0.017391304	72	0.01388889	0.030617678	0.083905416	0	0.016132426
Dziadówki	0.017391304	73	0.01369863	0.029777317	0.067734554	0	0.016104092
Tunel	0.017391304	74	0.01351351	0.028967254	0.051258581	0	0.015931199
Kozłów	0.017391304	75	0.01333333	0.028186275	0.034477498	0	0.015136737
Klimontów	0.017391304	76	0.01315789	0.027433206	0.017391304	0	0.012555618
Sędziszów	0.008695652	77	0.01298701	0.026706921	0	0	0.007100479
Kokotów	0.034782609	55	0.01818182	0.047286184	0.42299254	0	0.283343853
Węgrzyce Wielkie	0.043478261	54	0.01851852	0.047996661	0.510126975	0	0.206924503
Podłęże	0.034782609	53	0.01886792	0.048359966	0.493363844	0	0.165449498
Staniątki	0.034782609	52	0.01923077	0.048687553	0.495804729	0	0.148665856
Szarów	0.034782609	51	0.01960784	0.048977853	0.497940503	0	0.142231994
Kłaj	0.034782609	50	0.02	0.049229452	0.499771167	0	0.140071171
Stanisławice	0.034782609	49	0.02040816	0.049441101	0.50129672	0	0.139459953
Cikowice	0.034782609	48	0.02083333	0.049611734	0.502517162	0	0.139315411
Bochnia	0.034782609	47	0.0212766	0.049740484	0.503432494	0	0.139285836
Rzezawa	0.034782609	46	0.02173913	0.04982669	0.504042715	0	0.139274821
Jasień Brzeski	0.034782609	45	0.02222222	0.049869905	0.504347826	0	0.139236603
Brzesko Okocim	0.034782609	44	0.02272727	0.049869905	0.504347826	0	0.139042042

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Sterkowiec	0.034782609	43	0.02325581	0.04982669	0.504042715	0	0.138195769
Biadoliny	0.034782609	42	0.02380952	0.049740484	0.503432494	0	0.13515627
Bogumiłowice	0.034782609	41	0.02439024	0.049611734	0.502517162	0	0.126305838
Tarnów Mościce	0.034782609	40	0.025	0.049441101	0.50129672	0	0.105762186
Tarnów	0.026086957	39	0.02564103	0.049229452	0.499771167	0	0.068193054
Kłokowa	0.017391304	39	0.02564103	0.048977853	0.497940503	0	0.02847844
Łowczówek Pleśna	0.017391304	40	0.025	0.048687553	0.495804729	0	0.018609605
Łowczów	0.017391304	41	0.02439024	0.048359966	0.493363844	0	0.016545792
Tuchów	0.017391304	42	0.02380952	0.047996661	0.490617849	0	0.016192375
Lubaszowa	0.017391304	43	0.02325581	0.047599338	0.487566743	0	0.016142854
Siedliska k, Tuchowa	0.017391304	44	0.02272727	0.047169811	0.484210526	0	0.016137116
Chojnik	0.017391304	45	0.02222222	0.046709992	0.480549199	0	0.016136591
Gromnik	0.017391304	46	0.02173913	0.046221865	0.476582761	0	0.016136935
Bogoniowice Ciężkowice	0.017391304	47	0.0212766	0.045707472	0.472311213	0	0.016140658
Pławna	0.017391304	48	0.02083333	0.045168892	0.467734554	0	0.016169771
Bobowa	0.017391304	49	0.02040816	0.044608223	0.462852784	0	0.016350116
Bobowa Miasto	0.017391304	50	0.02	0.044027565	0.457665904	0	0.017202804
Jankowa	0.017391304	51	0.01960784	0.043429003	0.452173913	0	0.020144613
Wilczyska	0.026086957	52	0.01923077	0.042814594	0.495957285	0	0.027305129
Polna Szalowa	0.017391304	53	0.01886792	0.041426513	0.188558352	0	0.020144613
Wola Łużańska	0.017391304	54	0.01851852	0.040097629	0.174523265	0	0.017202804
Moszczenica	0.017391304	55	0.01818182	0.038825118	0.160183066	0	0.016350116
Małopolska	0.017391304	56	0.01785714	0.037606279	0.145537757	0	0.01616977
Gorlice	0.017391304	57	0.01754386	0.03643853	0.130587338	0	0.016140655
Zagórzany	0.017391304	58	0.01724138	0.03531941	0.115331808	0	0.016136895
Libusza	0.017391304	59	0.01694915	0.034246575	0.099771167	0	0.01613612
Biecz	0.017391304	60	0.01666667	0.033217793	0.083905416	0	0.016132428
Siepietnica	0.017391304	61	0.01639344	0.032230942	0.067734554	0	0.016104092
Skołoszyn	0.017391304	62	0.01612903	0.031284004	0.051258581	0	0.015931199
Przysieki	0.017391304	63	0.01587302	0.030375066	0.034477498	0	0.015136737
Jasło Niegłowice	0.017391304	64	0.015625	0.029502309	0.017391304	0	0.012555618
Jasło	0.008695652	65	0.01538462	0.028664008	0	0	0.007100479
Stróże	0.017391304	53	0.01886792	0.041787791	0.333180778	0	0.020144613
Grybów	0.017391304	54	0.01851852	0.040780142	0.322807018	0	0.017202804
Ptaszkowa	0.017391304	55	0.01818182	0.039792388	0.312128146	0	0.016350116
Mszalnica	0.017391304	56	0.01785714	0.038825118	0.301144165	0	0.016169771
Kamionka Wielka	0.017391304	57	0.01754386	0.037878788	0.289855072	0	0.016140657
Nowy Sącz Jamnica	0.017391304	58	0.01724138	0.036953728	0.27826087	0	0.016136931
Nowy Sącz	0.017391304	59	0.01694915	0.036050157	0.266361556	0	0.016136541
Nowy Sącz Biegonice	0.017391304	60	0.01666667	0.035168196	0.254157132	0	0.016136507

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Stary Sącz	0.017391304	61	0.01639344	0.034307876	0.241647597	0	0.016136505
Barcice	0.017391304	62	0.01612903	0.03346915	0.228832952	0	0.016136505
Rytro	0.017391304	63	0.01587302	0.032651902	0.215713196	0	0.016136505
Młodów	0.017391304	64	0.015625	0.031855956	0.20228833	0	0.016136505
Piwniczna Zdrój	0.017391304	65	0.01538462	0.031081081	0.188558352	0	0.016136505
Piwniczna	0.017391304	66	0.01515152	0.030327004	0.174523265	0	0.016136505
Łomnica Zdrój	0.017391304	67	0.01492537	0.029593412	0.160183066	0	0.016136505
Wierchomla Wielka	0.017391304	68	0.01470588	0.02887996	0.145537757	0	0.016136505
Zubrzyk	0.017391304	69	0.01449275	0.028186275	0.130587338	0	0.016136502
Żegiestów	0.017391304	70	0.01428571	0.027511962	0.115331808	0	0.016136468
Żegiestów Zdrój	0.017391304	71	0.01408451	0.026856609	0.099771167	0	0.016136084
Andrzejówka	0.017391304	72	0.01388889	0.02621979	0.083905416	0	0.016132426
Milik	0.017391304	73	0.01369863	0.025601069	0.067734554	0	0.016104092
Muszyna	0.017391304	74	0.01351351	0.025	0.051258581	0	0.015931199
Muszyna Zdrój	0.017391304	75	0.01333333	0.024416136	0.034477498	0	0.015136737
Powroźnik	0.017391304	76	0.01315789	0.023849025	0.017391304	0	0.012555618
Krynica Zdrój	0.008695652	77	0.01298701	0.023298217	0	0	0.007100479
Rudzice	0.017391304	55	0.01818182	0.046110666	0.086200102	0	0.055746749
Podgrabie	0.017391304	56	0.01785714	0.044367284	0.071103469	0	0.023521969
Podgrabie Wisła	0.017391304	57	0.01754386	0.043265613	0.058449483	0	0.017317161
Przylasek Rusiecki	0.017391304	58	0.01724138	0.042372881	0.049031307	0	0.016295364
Kraków Kościelniki	0.017391304	59	0.01694915	0.041696882	0.041327264	0	0.016155131
Kraków Nowa Huta Północ	0.017391304	60	0.01666667	0.041042113	0.034614831	0	0.016145452
Kraków Nowa Huta	0.017391304	61	0.01639344	0.04040759	0.028599793	0	0.01620388
Kraków Lubocza	0.017391304	62	0.01612903	0.039902845	0.023231716	0	0.016674282
Kraków Sambud	0.017391304	63	0.01587302	0.039410555	0.028088128	0	0.019875053
Dłubnia	0.017391304	62	0.01612903	0.040780142	0.043166793	0	0.039367297

Source: the author's own elaboration using the Gephi software.

Figures 6 and 7 present the distribution of exemplary parameters of individual network nodes for the network after the 1 modification. As in the previous drawings, the larger the node size and the darker the colour, the parameter value for a given node is larger.

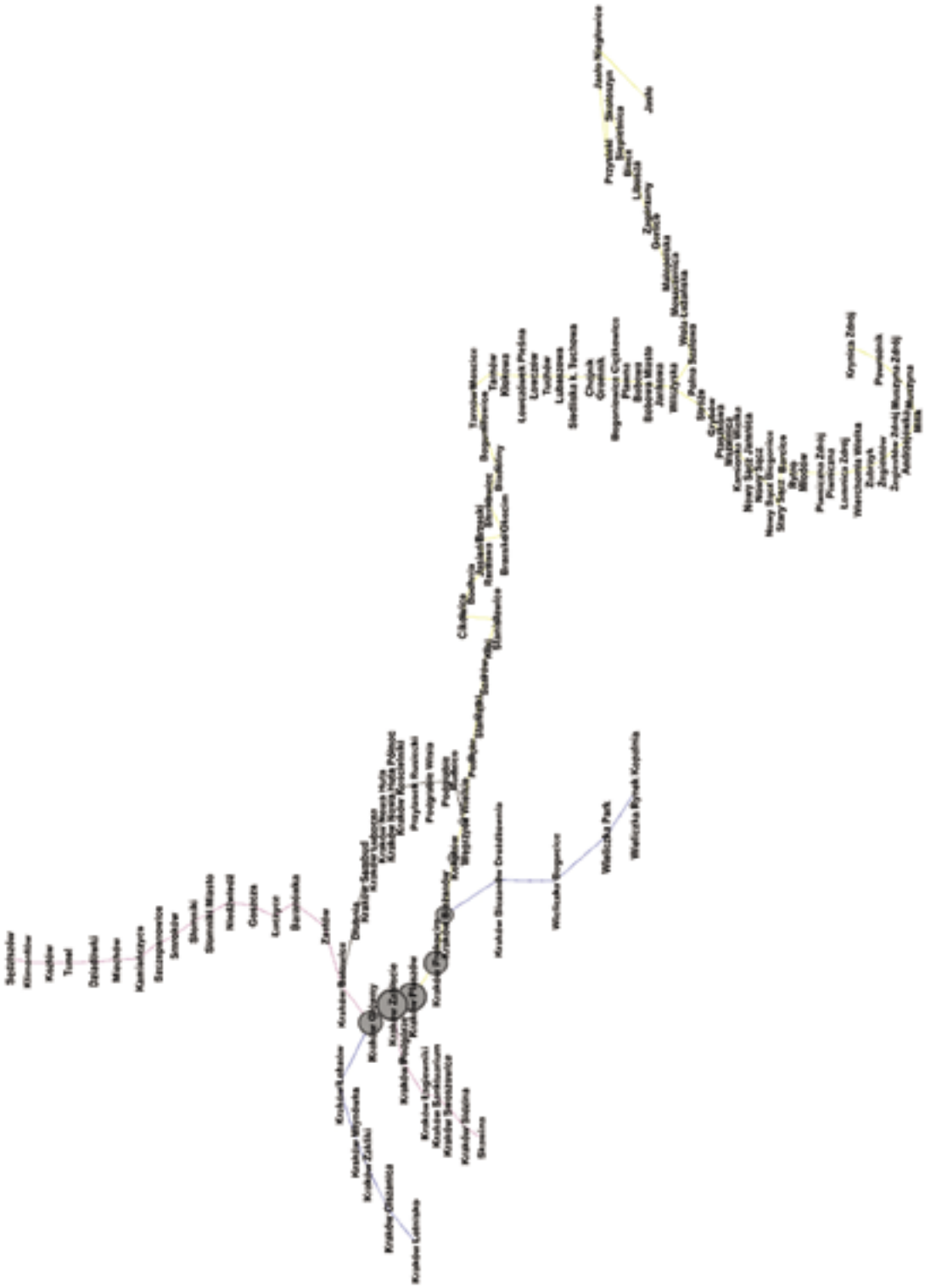
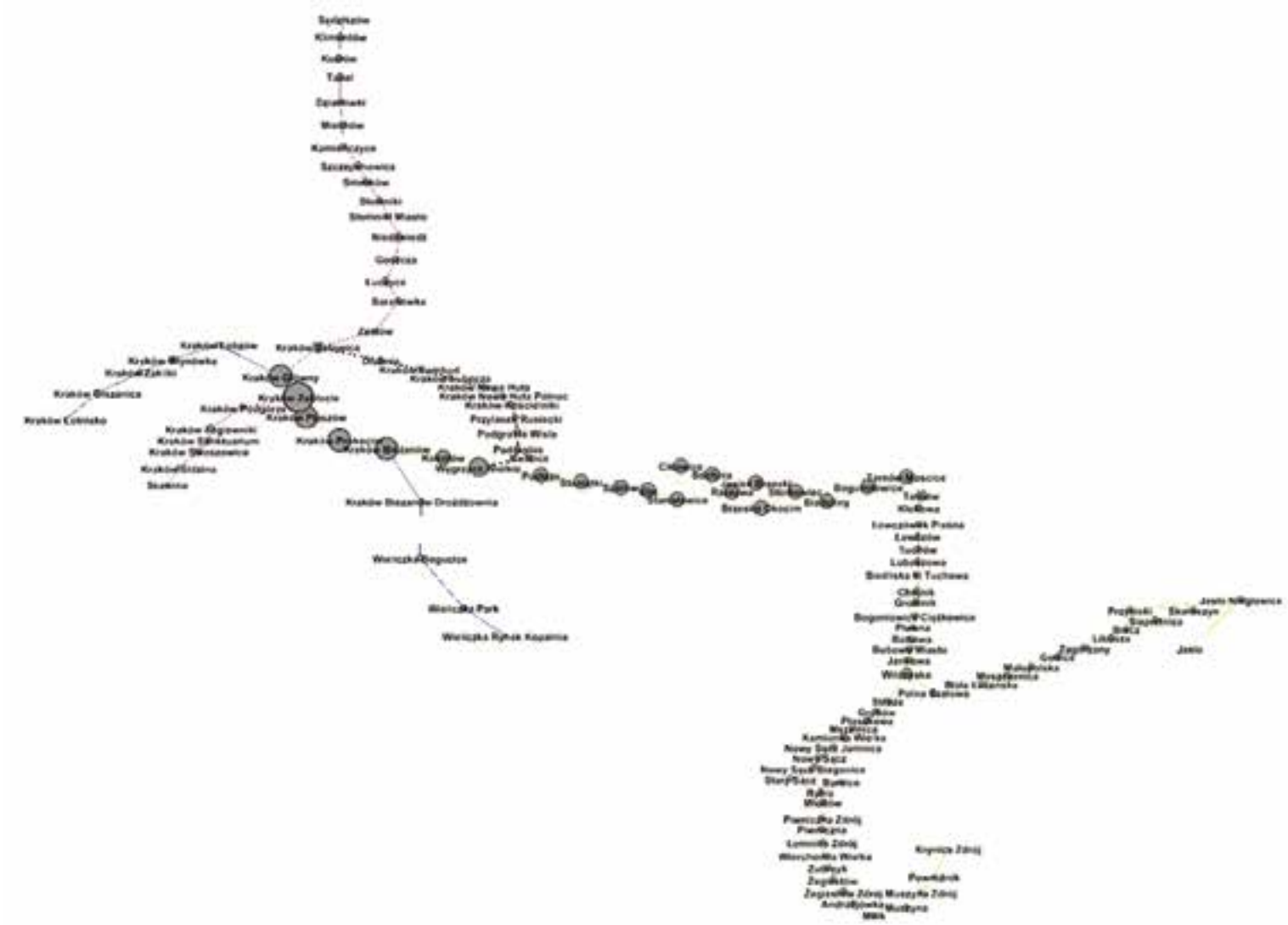


Figure 6. Eigencentrality for the analysed network after modification 1

Source: the author's own study using the Gephi software.

Figure 7. Degree for the analysed network after modification 1



Source: the author's own study using the Gephi software.

Then indicators for the entire network were calculated. Their values are presented in Table 5.

Table 5. Parameters of the analyzed network after 1 modification as a whole

Average Clustering Coefficent	Average path length	Diameter	Radius of a network	Average nodes degree
0	26.6347826	77	39	2.448276

Source: the author's own elaboration based on table 4.

As shown in Tables 4 and 5 and Figures 6 and 7 in the network after the modification, the Kraków Główny, Kraków Zabłocie and Kraków Płaszów stations still play the main role but the average path length has improved, which is a good result of the proposed modification, but unfortunately the average degree of the network node has deteriorated.

Next, the parameters for the network were calculated after the addition of modification 2, the results of the calculations of individual node parameters are presented in Table 6.

Table 6. Parameters of individual network nodes (stations) for the analysed network of Koleje Małopolskie connections after modification 2.

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Kraków Lotnisko	0.008547009	65	0.01538462	0.036369288	0	0	0.006813712
Kraków Zakliki	0.017094017	63	0.01587302	0.039169736	0.03389331	0	0.017925986
Kraków Olszanica	0.017094017	64	0.015625	0.037729765	0.017094017	0	0.012442416
Kraków Młynówka	0.017094017	62	0.01612903	0.040695652	0.050397878	0	0.037877347
Kraków Łobzów	0.025641026	61	0.01639344	0.042314647	0.070291777	0	0.150873621
Kraków Główny	0.051282051	60	0.01666667	0.043984962	0.332139604	0	0.734037724
Kraków Płaszów	0.051282051	58	0.01724138	0.045685279	0.390666677	0	0.862267729
Kraków Prokocim	0.051282051	57	0.01754386	0.046447003	0.39685589	0	0.693278835
Kraków Bieżanów	0.051282051	56	0.01785714	0.047234558	0.447169561	0	0.479516346
Kraków Bieżanów Drożdżownia	0.017094017	57	0.01754386	0.045226131	0.050397878	0	0.095942883
Wieliczka Bogucice	0.017094017	58	0.01724138	0.043349389	0.03389331	0	0.027398187
Wieliczka Park	0.017094017	59	0.01694915	0.041592606	0.017094017	0	0.013808996
Wieliczka Rynek Kopalnia	0.008547009	60	0.01666667	0.039945374	0	0	0.006982181
Kraków Zabłocie	0.076923077	59	0.01694915	0.044948137	0.415355111	0	1
Skawina	0.008547009	65	0.01538462	0.035790762	0	0	0.006757824
Kraków Sidzina	0.017094017	64	0.015625	0.037107517	0.017094017	0	0.011989969
Kraków Swoszowice	0.017094017	63	0.01587302	0.038499506	0.03389331	0	0.01479442
Kraków Sanktuarium	0.017094017	62	0.01612903	0.039972668	0.050397878	0	0.018640885
Kraków Łagiewniki	0.017094017	61	0.01639344	0.041533546	0.066607722	0	0.040792232
Kraków Podgórze	0.017094017	60	0.01666667	0.043189369	0.082522841	0	0.179788481
Kraków Batowice	0.025641026	61	0.01639344	0.042888563	0.27747605	0	0.147357028
Zastów	0.017094017	62	0.01612903	0.041577825	0.225464191	0	0.03763224
Baranówka	0.017094017	63	0.01587302	0.040317023	0.212496316	0	0.018795096

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Łuczyce	0.017094017	64	0.015625	0.039104278	0.199233716	0	0.015772479
Goszcza	0.017094017	65	0.01538462	0.037937743	0.185676393	0	0.015339375
Niedźwiedź	0.017094017	66	0.01515152	0.036815607	0.171824344	0	0.015285532
Słomniki Miasto	0.017094017	67	0.01492537	0.035736103	0.157677571	0	0.015279787
Słomniki	0.017094017	68	0.01470588	0.034697509	0.143236074	0	0.015279261
Smroków	0.017094017	69	0.01449275	0.033698157	0.128499853	0	0.015279217
Szczepanowice	0.017094017	70	0.01428571	0.03273643	0.113468907	0	0.015279186
Kamieńczyce	0.017094017	71	0.01408451	0.031810767	0.098143236	0	0.015278861
Miechów	0.017094017	72	0.01388889	0.030919662	0.082522841	0	0.015275703
Dziadówki	0.017094017	73	0.01369863	0.030061665	0.066607722	0	0.01525075
Tunel	0.017094017	74	0.01351351	0.029235382	0.050397878	0	0.015095255
Kozłów	0.017094017	75	0.01333333	0.028439475	0.03389331	0	0.014365306
Klimontów	0.017094017	76	0.01315789	0.027672658	0.017094017	0	0.011945164
Sędziszów	0.008547009	77	0.01298701	0.026933702	0	0	0.00675381
Kokotów	0.034188034	55	0.01818182	0.047891936	0.429661124	0	0.25792823
Węgrzyce Wielkie	0.042735043	54	0.01851852	0.04856787	0.513534719	0	0.188232396
Podłęże	0.034188034	53	0.01886792	0.048892603	0.49602122	0	0.150760597
Staniątki	0.034188034	52	0.01923077	0.049180328	0.498084291	0	0.135684846
Szarów	0.034188034	51	0.01960784	0.049429658	0.499852638	0	0.129948551
Kłaj	0.034188034	50	0.02	0.049639372	0.50132626	0	0.128044081
Stanisławice	0.034188034	49	0.02040816	0.049808429	0.502505158	0	0.127513165
Cikowice	0.034188034	48	0.02083333	0.04993598	0.503389331	0	0.127389626
Bochnia	0.034188034	47	0.0212766	0.050021377	0.50397878	0	0.127364792
Rzezawa	0.034188034	46	0.02173913	0.050064185	0.504273504	0	0.127355702
Jasień Brzeski	0.034188034	45	0.02222222	0.050064185	0.504273504	0	0.127323694
Brzesko Okocim	0.034188034	44	0.02272727	0.050021377	0.50397878	0	0.1271579
Sterkowiec	0.034188034	43	0.02325581	0.04993598	0.503389331	0	0.126424528
Biadoliny	0.034188034	42	0.02380952	0.049808429	0.502505158	0	0.123747152
Bogumiłowice	0.034188034	41	0.02439024	0.049639372	0.50132626	0	0.115831956
Tarnów Mościce	0.034188034	40	0.025	0.049429658	0.499852638	0	0.09721996
Tarnów	0.025641026	39	0.02564103	0.049180328	0.498084291	0	0.062859123
Kłokowa	0.017094017	39	0.02564103	0.048892603	0.49602122	0	0.026473535
Łowczówek Pleśna	0.017094017	40	0.025	0.04856787	0.493663425	0	0.017495364
Łowczów	0.017094017	41	0.02439024	0.048207664	0.491010905	0	0.015640511
Tuchów	0.017094017	42	0.02380952	0.047813649	0.48806366	0	0.015327728
Lubaszowa	0.017094017	43	0.02325581	0.047387606	0.484821692	0	0.015284636
Siedliska k, Tuchowa	0.017094017	44	0.02272727	0.046931408	0.481284999	0	0.015279729
Chojnik	0.017094017	45	0.02222222	0.046447003	0.477453581	0	0.015279288
Gromnik	0.017094017	46	0.02173913	0.045936396	0.473327439	0	0.015279579
Bogoniowice Ciężkowice	0.017094017	47	0.0212766	0.04540163	0.468906572	0	0.015282791

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Pławna	0.017094017	48	0.02083333	0.044844768	0.464190981	0	0.015308403
Bobowa	0.017094017	49	0.02040816	0.044267877	0.459180666	0	0.015470326
Bobowa Miasto	0.017094017	50	0.02	0.043673012	0.453875626	0	0.0162515
Jankowa	0.017094017	51	0.01960784	0.043062201	0.448275862	0	0.018995489
Wilczyska	0.025641026	52	0.01923077	0.042437432	0.490274094	0	0.02574919
Polna Szalowa	0.017094017	53	0.01886792	0.041067041	0.185676393	0	0.018995489
Wola Łużańska	0.017094017	54	0.01851852	0.039755352	0.171824344	0	0.0162515
Moszczenica	0.017094017	55	0.01818182	0.038499506	0.157677571	0	0.015470326
Małopolska	0.017094017	56	0.01785714	0.03729678	0.143236074	0	0.015308403
Gorlice	0.017094017	57	0.01754386	0.036144578	0.128499853	0	0.015282788
Zagórzany	0.017094017	58	0.01724138	0.035040431	0.113468907	0	0.015279546
Libusza	0.017094017	59	0.01694915	0.033981992	0.098143236	0	0.015278891
Biecz	0.017094017	60	0.01666667	0.032967033	0.082522841	0	0.015275706
Siepietnica	0.017094017	61	0.01639344	0.031993437	0.066607722	0	0.01525075
Skołoszyn	0.017094017	62	0.01612903	0.031059198	0.050397878	0	0.015095255
Przysieki	0.017094017	63	0.01587302	0.030162413	0.03389331	0	0.014365306
Jasło Niegłowice	0.017094017	64	0.015625	0.029301277	0.017094017	0	0.011945164
Jasło	0.008547009	65	0.01538462	0.028474081	0	0	0.00675381
Stróże	0.017094017	53	0.01886792	0.041415929	0.328912467	0	0.018995489
Grybów	0.017094017	54	0.01851852	0.040414508	0.318597112	0	0.0162515
Ptaszkowa	0.017094017	55	0.01818182	0.039433771	0.307987032	0	0.015470326
Mszalnica	0.017094017	56	0.01785714	0.038474186	0.297082228	0	0.015308403
Kamionka Wielka	0.017094017	57	0.01754386	0.037536092	0.2858827	0	0.01528279
Nowy Sącz Jamnica	0.017094017	58	0.01724138	0.036619718	0.274388447	0	0.015279577
Nowy Sącz	0.017094017	59	0.01694915	0.035725191	0.262599469	0	0.015279247
Nowy Sącz Biegonice	0.017094017	60	0.01666667	0.034852547	0.250515768	0	0.015279219
Stary Sącz	0.017094017	61	0.01639344	0.034001744	0.238137342	0	0.015279216
Barcice	0.017094017	62	0.01612903	0.033172668	0.225464191	0	0.015279216
Rytro	0.017094017	63	0.01587302	0.032365145	0.212496316	0	0.015279216
Młodów	0.017094017	64	0.015625	0.031578947	0.199233716	0	0.015279216
Piwniczna Zdrój	0.017094017	65	0.01538462	0.0308138	0.185676393	0	0.015279216
Piwniczna	0.017094017	66	0.01515152	0.030069391	0.171824344	0	0.015279216
Łomnica Zdrój	0.017094017	67	0.01492537	0.029345372	0.157677571	0	0.015279216
Wierchomla Wielka	0.017094017	68	0.01470588	0.028641371	0.143236074	0	0.015279216
Zubrzyk	0.017094017	69	0.01449275	0.027956989	0.128499853	0	0.015279214
Żegiestów	0.017094017	70	0.01428571	0.027291812	0.113468907	0	0.015279186
Żegiestów Zdrój	0.017094017	71	0.01408451	0.026645411	0.098143236	0	0.015278861
Andrzejówka	0.017094017	72	0.01388889	0.026017345	0.082522841	0	0.015275703
Milik	0.017094017	73	0.01369863	0.025407166	0.066607722	0	0.01525075
Muszyna	0.017094017	74	0.01351351	0.024814422	0.050397878	0	0.015095255

City (Node)	Normalized degree	Eccentricity	Radius	Closeness centrality	Betweeness centrality	Clustering	Eigencentrality
Muszyna Zdrój	0.017094017	75	0.01333333	0.024238658	0.03389331	0	0.014365306
Powroźnik	0.017094017	76	0.01315789	0.023679417	0.017094017	0	0.011945164
Krynica Zdrój	0.008547009	77	0.01298701	0.023136247	0	0	0.00675381
Rudzice	0.017094017	55	0.01818182	0.046632124	0.083997341	0	0.050984687
Podgrabie	0.017094017	56	0.01785714	0.044844768	0.069119883	0	0.021872706
Podgrabie Wisła	0.017094017	57	0.01754386	0.043705641	0.056601922	0	0.016319424
Przylasek Rusiecki	0.017094017	58	0.01724138	0.04279444	0.047362248	0	0.015417038
Kraków Kościelniki	0.017094017	59	0.01694915	0.042146974	0.040073082	0	0.015295114
Kraków Nowa Huta Północ	0.017094017	60	0.01666667	0.041518808	0.033883868	0	0.015286975
Kraków Nowa Huta	0.017094017	61	0.01639344	0.040909091	0.028368311	0	0.015339492
Kraków Lubocza	0.017094017	62	0.01612903	0.040428473	0.023477691	0	0.015772487
Kraków Sambud	0.017094017	63	0.01587302	0.039959016	0.028463512	0	0.018795096
Dłubnia	0.017094017	62	0.01612903	0.041371994	0.043323614	0	0.03763224
Kraków Olsza	0.017094017	61	0.01639344	0.04189044	7.37E-04	0	0.063163892
Kraków Dąbie	0.017094017	60	0.01666667	0.043141593	0.012673151	0	0.183247331

Source: the author's own elaboration using the Gephi software.

The figures show the distribution of the exemplary parameters of individual network nodes for the network after the 2nd modification. As in the previous drawings, the larger the node size and the darker the colour, the parameter value for the given node is greater.

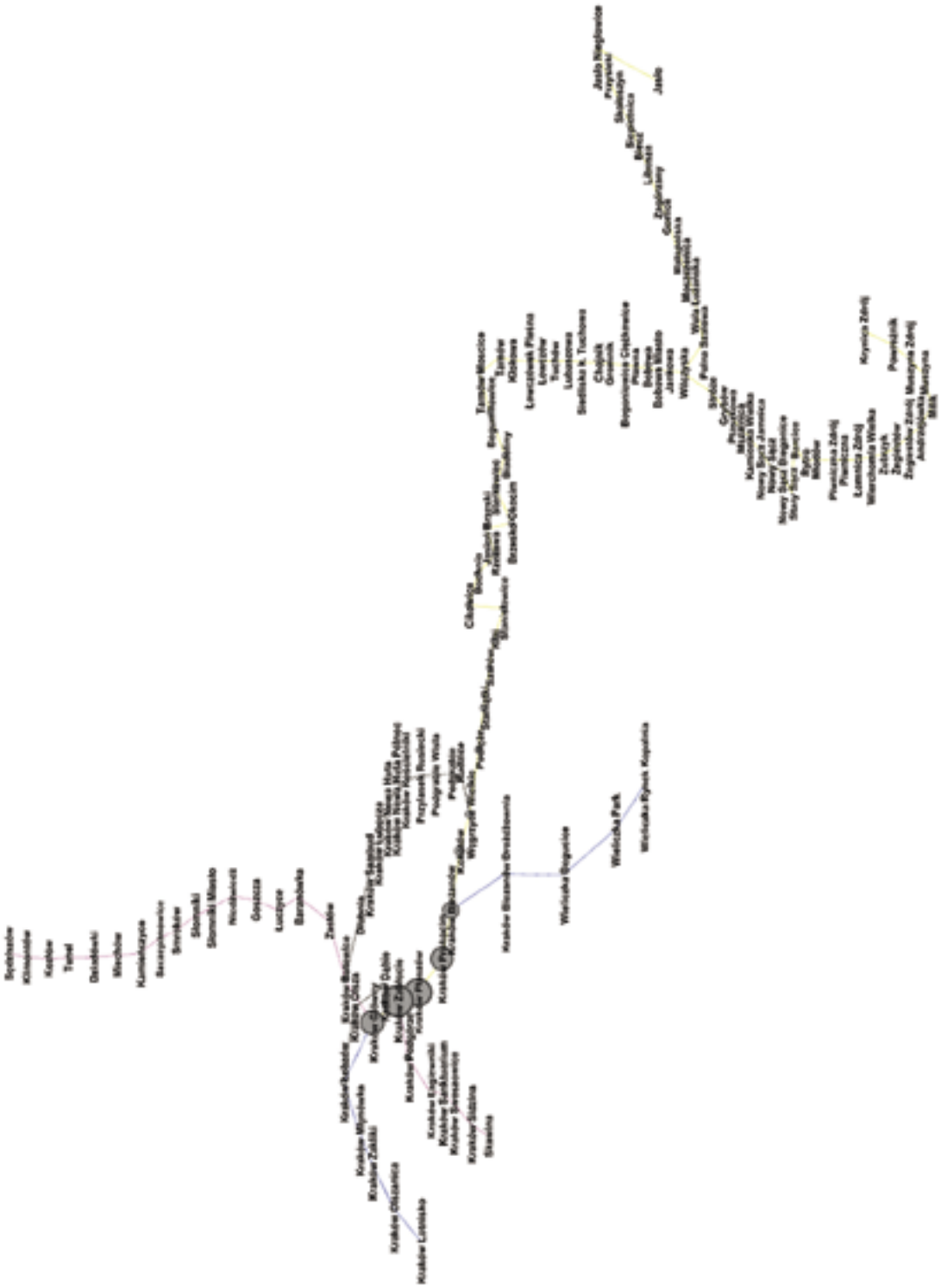
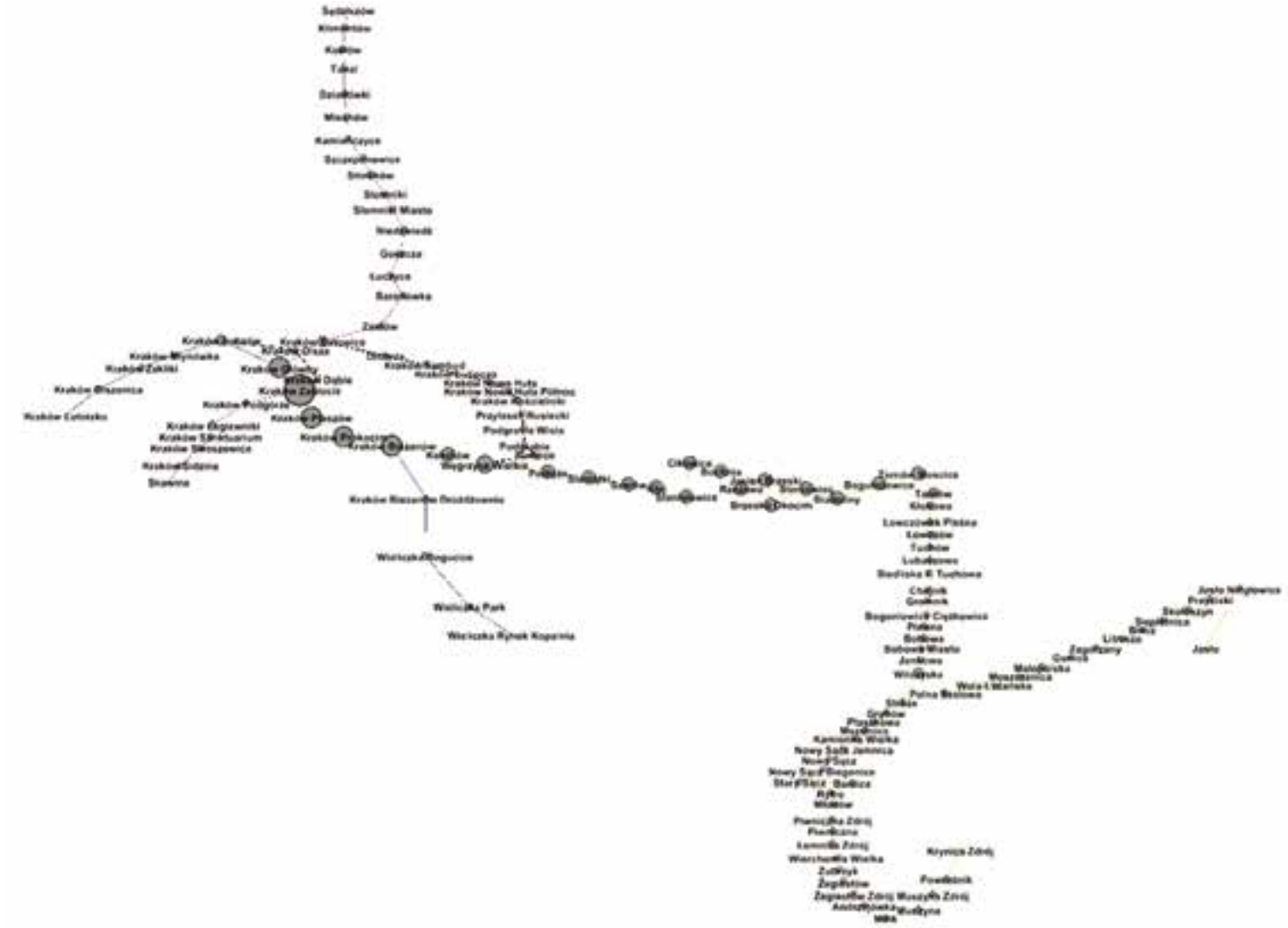


Figure 8. Eigencentrality for the analysed network after modification 2

Source: the author's own study using the Gephi software.

Figure 9. Degree for the analysed network after modification 2



Source: the author's own study using the Gephi software.

Then the indicators for the entire network were calculated. Their values are presented in Table 7.

Table 7. Parameters of the analyzed network after modification 2 as a whole

Average Clustering Coefficent	Average path length	Diameter	Radius of a network	Average nodes degree
0	26.53310155	77	39	2.457627119

Source: the author's own elaboration based on table 6.

As shown in Tables 6 and 7 and Figures 8 and 9, the introduced modification did not change the validity of individual network nodes but contributed to a slight improvement in the average path length.

In order to better illustrate the effect of the proposed changes in the network parameters, table 8 presents the parameters of the original network and networks with the proposed modifications.

Table 8. Comparison of the parameters of the analyzed network before and after the modifications

	Average Clustering Coefficent	Average path length	Diameter	Radius of a network	Average nodes degree
Original network	0	27.01437556	77	39	2.471698113
Network after modification 1	0	26.6347826	77	39	2.448276
Network after modification 2	0	26.53310155	77	39	2.457627119
The effect of modification	No parameter changes	Improvement of the parameter	No parameter changes	No parameter changes	Deterioration of the parameter

Source: the author's own elaboration based on tables 3, 5 and 7.

As shown in Table 8, the modifications had a positive impact on the change in the average path length, so the implementation of the proposed modification to a small extent contributes to the improvement of its parameters, which in the future could translate into an increase in the number of travelers and consequently have a positive impact on the financial results of the company. The introduced modifications also increase the transport accessibility of the carrier's transport network, which is important in terms of providing a better level of customer service, and this is one of the most important elements of managing a service enterprise [Witkowski 1995].

Summary

The analysed network of connections of Koleje Małopolskie is intended to enable the implementation of efficient and useful connections in the area of the Kraków agglomeration and the entire voivodship. The analyses carried out using the graph theory showed that the

network enables communication on the main relations in the voivodship (especially within the Kraków agglomeration). The proposed modifications do not significantly affect the improvement of network parameters, but they also include proposals for connections via highly urbanized and populated areas of Kraków (including Nowa Huta).

The introduction of these connections may contribute to improving the carrier's offer, which may translate into changing the transport habits of at least some residents (change of means of transport from an individual vehicle to public transport) and what will be very important is the availability of transport in the analysed area. It may contribute to very important and necessary changes related to, among others, reduced congestion in the city and the emission of harmful substances. An important element of such a process will also increase the importance of the carrier on the local market of public transport services and it will probably cause a further increase in the number of travelers, which should also translate into the company's financial results and strengthening its market position.

The above proposal to add new nodes to the existing connection network may contribute to the competitiveness of the carrier, as well as the entire network of its connections will then improve (which has been demonstrated as part of the analyses and simulations and the results obtained, among others, a slight improvement of the average path length).

According to the author, the Koleje Małopolskie railway carrier should consider the implementation of the analyses into its activities related to investments or development of the network, e.g. using the graphs used in the article. This will allow obtaining additional information about the parameters of the proposed network (or its modification), which information may contribute to reducing the risk of introducing negative changes in the network of connections.

Undoubtedly, the conducted and proposed analysis of connection networks using the graph theory does not exhaust the issue and the factors necessary to be taken into account while managing the company when making key decisions. The afore-mentioned series of factors (including technical parameters of individual connection sections between nodes) are not taken into account, however, they were not the purpose of the article. As it was shown, the aim of the article was to analyse the carrier's connection network and propose this analysis as another element that is part of the process of making key decisions in the company.

References

1. Amaral LAN, Scala A, Barthelemy M, Stanley HE., 1997. Classes of small-world net- works. *Proc Natl Acad Sci* USA2000; (21):11149–52.
2. Arenas A., Danon L., Diaz-Guilera A., Gleiser P.M., Guimera R., 2003. Community analysis in social networks. *Eur Phys JB*, Vol. 38(2), pp. 373–80.
3. Bullmore E., Sporns O., 2009. Complex brain networks: graph theoretical analysis of structural and functional systems. *Nat Rev Neuro Sci*, vol. 10(3), pp. 186–98.

4. Dunn S., Wilkinson S., 2017. Hazard tolerance of spatially distributed complex networks. *Reliability Engineering and System Safety* 157 1–12, Elsevier.
5. Li H, Guo XM, Xu Z, Hu XB., 2014. A study on the spatial vulnerability of the civil aviation network system in China, Qingdao: *Proceedings of the IEEE 17th international conference on intelligent transportation systems*, China.
6. Newman M.E.J., 2010. *Networks: An Introduction*. New York: Oxford University Press Inc.
7. Newman MEJ, Watts DJ, Strogatz SH., 2002. Random graph models of social net- works. *Proc Natl Acad Sci* USA, 99, 2566–72.
8. Ouyang M, Pan Z, Hong L, He Y., 2015. Vulnerability analysis of complementary transportation systems with applications to railway and airline systems in China. *Reliab Eng Syst Saf*, 142:248.
9. Rual J.-F., Venkatesan K., Hao T., Hirozane-Kishikawa T., Dricot A., Li N., Berriz G.F., Gibbons F.D., Dreze M., Ayivi-Guedehoussou N., Klitgord N., Simon C., Boxem M., Milstein S., Rosenberg J., Goldberg D.S., Zhang L.V., Wong S.L., Franklin G., Li S., Albala J.S., Lim J., Fraughton C., Llamosas E., Cevik S., Bex C., Lamesch P., Sikorski R.S., Vandenhaute J., Zoghbi H.Y., Smolyar A., Bosak S., Sequerra R., Doucette-Stamm L., Cusick M.E., Hill D.E., Roth F.P., Vidal M., 2005. Towards a proteome-scale map of the human protein – protein interaction network. *Nature,* Vol. 437(7062).
10. Sporns O., 2002. Network analysis, complexity, and brain function. *Complexity,* Vol. 8(1), pp. 56–60.
11. Stam C.J., Reijneveld J.C., 2007. Graph theoretical analysis of complex networks in the brain. *Nonlinear Biomed Phys*, Vol. 1(3), pp. 1–19.
12. Tarapata Z., 2015. Modelling and analysis of transportation networks using complex networks: Poland case study. *The Archives of Transport*, Vol. 36, Issue 4.
13. Valverde S., Solé R.V., 2003. Hierarchical small worlds in software architecture. *Arxiv Prepr Cond-Mat*/0307278.
14. Wilkinson S, Dunn S, Ma S., 2012. The vulnerability of the European air traffic network to spatial hazards. *Nat Hazards*, 60(3), 1027–36.
15. Witkowski J., 1995. *Strategia logistyczna przedsiębiorstw przemysłowych*. Wrocław: Wydawnictwo Akademii Ekonomicznej we Wrocławiu.
16. malopolskiekoleje.pl/ [accessed: 28.03.2018].
17. Statistical data received electronically from the Office of Rail Transport on January 16, 2018.