Marcin Łupiński[±]

On the Usefulness of Financial Variables Realised Volatility for Recession Forecasting and Business Cycles Turning Points Dating

Abstract

The main goal of this paper was to check usefulness of introducing measures of the financial markets risk into multivariate forecasting and business cycle dating models to improve their predictive and turning points detection power. Realised volatility was selected as market risk synthetic measure and introduced into two recession dating algorithms: Harding & Pagan (2002) mechanical procedure and Markov Switching Dynamic Factor Model (MS-DFM) with mixed frequencies and missing data handling. In the theoretical part of the article mathematical background of the realised volatility concept and MS-DFM model were presented. It was also described how the output of the MS-DFM model can be used to date turning points. This approach to local maxima detection was compared with Harding and Pagan competitor algorithm. In the practical part of the paper recession detection improvements stemming from introduction of realised volatility measures into MS-DFM model/Harding & Pagan procedure were examined for US and four Western Europe countries (Germany, France, United Kingdom and Italy) in the time span of 20 years between 1990 and 2010.

Keywords: realised volatility, Markov switching model, dynamic factor model, turning points detection

JEL classification: C32, E32, E37, G17

[±] University of Warsaw and National Bank of Poland

1. Introduction

This paper was motivated with poor performance of econometric models used for short-term forecasting and turning points (TPs) detection during the Great Moderation of 2008-2010. Due to this fact there is a constantly growing interest in applying selected financial variables to improve applied models prediction and business cycle dating power. Beside that there is a need of quantifying impact of risk associated with economic agents' financial activity on the real sphere of the economies. Availability of multivariate business cycle analysis methods allows to perform nonlinear multi-factor turning points detection based both on macroeconomic and financial data. One of the best performing models described in the literature is the Markov Switching Dynamic Factor Model (MS-DFM) applied also in this paper.

The goal of this article is to compute realised volatility (RV) measures quantifying risk associated with financial activity of economic agents caused by shocks affecting economies and then apply these measures in Markov Switching Dynamic Factor Models with mixed-frequencies and missing data handling to detect business cycle turning points of US and four Western Europe developed countries (Germany, France, UK and Italy). Moreover the paper is aimed at checking improvement of TPs' dating models performance (speed, accuracy) with a measure of financial risk included in the group of input variables in a real-time exercise.

2. Literature overview

There is vast literature concerning realised volatility measure theory and its practical computations. The good example of such literature is Andersen, Bollerslev *et al*. series of papers (2000, 2001, 2003, 2004, 2007) which introduced the mentioned concept and mastered methods of its estimation. Meanwhile Corsi *et al*. (2008) proposed their own approach to RV estimation applying Heterogeneous Autoregressive Models (HAR approach). Barndorff-Nielsen *et al*. (2004, 2005) in their two consecutive papers decomposed RV into continuous and jump component with bipower variation.

Univariate Markov switching (MS) models were introduced to economics and finance by Hamilton (1988, 1989) due to observed stylised facts of periodical nonlinear changes in macroeconomics and market data behaviour. Multivariate dynamic factor models (DFMs) were applied by Stock & Watson (1989) and then popularised in the series of their seminal works (eg 1998). Combination of multivariate Markov switching and DFMs (MS-DFMs) was firstly introduced by Chauvet (1998). After that Chauvet & Hamilton (2005) presented MS-DFMs practical application in business cycle turing points dating procedure. Recently Camacho & Pérez-Quirós (2009) showed application of MS-DFMs with mixed frequency and missing data handling for business cycle analysis and forecasting. This augmentation is taken into account in the presented model. Finally Chauvet & Piger (2008) compared MS-DFMs with alternative TP dating algorithms.

One of the best-performing methods compared in this work with MS-DFMs was Harding & Pagan TPs' dating algorithm (2002), which is also used as a benchmark detection procedure in this article.

3. Research components

3.1. Realised volatility concept (RV)

The mathematical background of realized volatility description starts with standard stochastic differential equation used for modeling logarithmic asset price as continuous-time jump diffusion process:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t), 0 \leq t \leq T, \quad (1)$$

where $p(t)$ is a logarithmic asset price, $\mu(t)$ is continuous and locally bounded mean process, $\sigma(t)$ is continuous and strictly positive volatility process, $q(t)$ is counting process and $\kappa(t)$ determines discrete jumps of $p(t)$. Quadratic variation for cumulative return $(r(t) = p(t) - p(0))$ of the process defined above comes from the continuous part and squared jumps recorded during considered time span $[0, t]$:

$$[r, r]_t = \int_0^t \sigma^2(s)ds + \sum_{0<s\leq t} \kappa^2(s). \quad (2)$$

Realised volatility (RV) is computed as a sum of squared price returns recorded in intradaily periods of time lasting *m*:

$$RV(m)_{t+1} = \sqrt{\sum_{i=1}^{(1/m)} r_{t+im,m}^2}. \quad (3)$$

For $m \to 0$ RV converges uniformly in probability to the contribution of quadratic variation registered in the time span $[t, t+1]$:

$$RV(m)_{t+1} \to \int_t^{t+1} \sigma^2(s)ds + \sum_{t<s\leq t+1} \kappa^2(s). \quad (4)$$

Due to RV's dual characteristics Andersen *et al*. (2004) suggested that its two main components (continuous and jump) should be modeled separately. To separate components of RV we follow Barndorff-Nielsen's (2004) idea of the bipower variation:

$$BV(m)_{t+1} = \sqrt{\mu_1^{-2} \sum_{i=2}^{(1/m)} \left|r_{t+im,m}^2\right|\left|r_{t+(i-1)m,m}^2\right|},\ \mu_1 = \sqrt{\frac{2}{\pi}}. \tag{5}$$

It converges uniformly in probability to the continuous part of RV:

$$BV(m)_{t+1} \to \int_t^{t+1} \sigma^2\,(d)ds. \tag{6}$$

The residual part determines changes of asset's prices caused by jumps associated mainly with shocks affecting an analysed economy (they are left bounded to 0):

$$J(m)_{t+1} = max[RV(m)_{t+1} - BV(m)_{t+1}, 0], \tag{7}$$

where

$$VM(m)_{t+1} - BV(m)_{t+1} \to \sum_{t<s\leq t+1} \kappa^2(s). \tag{8}$$

From econometric point of view RV can be modeled with the extended Heterogeneous ARCH model (HAR-RV) proposed by Corsi *et al*. (2008) as a linear function of the lagged realised volatilities over different horizons (daily, weekly and monthly respectively):

$$RV_{t+1}^m = \beta_0 + \beta_D RV_t^m + \beta_W RV_{t-5,t}^m + \beta_M RV_{t-22,t}^m + \varepsilon_{t+1}. \tag{9}$$

The model above is able to replicate long memory of realised volatility measured with slowly decaying autocorrelation. Jump component is introduced to the framework (HAR-RV-J) with an additional variable computed with nonparametric measure derived on the previous slide:

$$RV_{t+1}^m = \beta_0 + \beta_D RV_t^m + \beta_W RV_{t-5,t}^m + \beta_M RV_{t-22,t}^m + B_1 J_t + \varepsilon_{t+1}. \tag{10}$$

3.2. Markov-switching DFMs with moxed frequencies and missing data handling

Mixed frequencies problem arises when quarterly GDP is combined within one model with monthly macroeconomic indicators like industrial production (IP) or CPI time series. Approximation of proposed variables growth rates proposed by Mariano & Murasawa (2003) is used to combine them in one equation:

$$q_t = \frac{1}{3}m_t + \frac{2}{3}m_{t-1} + m_{t-2} + \frac{2}{3}m_{t-3} + \frac{1}{3}m_{t-4}. \tag{11}$$

Missing data problem is a consequence of modeling within one analytical structure mixed frequencies and different deadlines of particular time series data publication (ragged edges). To solve this issue the other idea of Mariano & Murasawa was applied. Namely, observations which were not available were replaced with draws from Gaussian distribution $\left(\theta_t \sim N\left(0, \sigma_\theta^2\right)\right)$.

Markov Switching Dynamic Factor Model (MS-DFM) decomposes a group of input time series into two different components: common factors and idiosyncratic ones. The last ones are responsible for individual behaviour of included series. To model regime changes in the business cycle intercept and variance switching approach is chosen:

$$f_t = \alpha s_t + \sum_{i=1}^{k} \alpha_{1,i} f_{t-i} + \sigma s_t \epsilon_t^f. \tag{12}$$

The state variable S_t follows the regime-switching Markov process with probabilities of transition between states equal to $P(S_t = j | S_{t-1} = i) = p_{i.j}$.

The general MS-DFM model structure can be described as:

$$\begin{bmatrix} y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \beta_1\left(\frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4}\right) \\ \beta_2 f_t \end{bmatrix} + \begin{bmatrix} \frac{1}{3}u_{1,t} + \frac{2}{3}u_{1,t-1} + u_{1,t-2} + \frac{2}{3}u_{1,t-3} + \frac{1}{3}u_{1,t-4} \\ V_t \end{bmatrix}, \tag{13}$$

where β_2 groups the factor loading of l monthly time series and V_t their idiosyncratic components. The components of the matrix V_t evolve in accordance with the formula:

$$v_{j,t} = \sum_{i=1}^{k} a_{j,i} f_{t-i} + \epsilon_t^{v_j}, \epsilon_t^{v_j} \sim i.i.d.N\left(0, \sigma_{v_j}^2\right). \tag{14}$$

Described model is estimated with two main methods:

- Approximated Maximum Likelihood Estimation (MLE) with nonlinear Kalman filter described by Kim (1994) and Chauvet (1998);
- Gibbs sampling (Bayesian Monte Carlo Markov Cahin (MCMC) algorithm presented by Kim & Nelson (1999)).

The choice of Chauvet & Piger (2008) and Camacho & Pérez-Quirós (2009) was selected in this paper and combined MLE and nonlinear Kalman filter method is applied to estimate MS-DFM model. MS-DFM model's structure is identified with adjusted likelihood ratio (LR) test for a number of regimes. For this test Garcia & Perron (1996) computed upper bounds of *p*-values and Ang & Bekaert (2002) determined distribution of mentioned test statistics. The lag structure of the model dynamics is chosen with Akaike and Bayesian information criterions (AIC, BIC). Two following tables show results of LR test of the MS-DFM model structure for the five surveyed countries.

Table 1. LR test 2 vs. 3 regimes

Country	US	Germany	France	UK	Italy
LR	33.6814	37.9379	55.9152	45.4553	49.8167
df	5	9	10	5	6

Source: own computations.

In Table 2 sample results of AR structure identification procedure with AIC and BIC were depicted.

Table 2. IC US example

Factor AR structure	AIC	BIC
AR(3)	-8.9411	-8.6934
AR(4)	-8.9152	-8.6793

Source: own computations.

The MS-DFM's turning points dating algorithm used for the practical part consists of two rules: one for a local peak and one for a local trough:

1. Peak at time t if $Pr[S_{t-1} = 0|I_{t-1}] < 0.5$ *and* $Pr[S_t = 0|I_t] \geq 0.5$ *and* $Pr[S_{t+1}|I_{t+1}] \geq 0.5$.
2. Trough at time t if $Pr[S_{t-1} = 0|I_{t-1}] \geq 0.5$ *and* $Pr[S_t = 0|I_t] < 0.5$ *and* $Pr[S_{t+1}|I_{t+1}] < 0.5$.

The results of this algorithm were compared with Harding & Pagan procedure output, presented in the paragraph below.

3.3. Harding and Pagan TPs dating algorithm

The Harding & Pagan TPs dating algorithm is a mechanical method which allows to extract common turning points from the set of time series. Steps of the HP TPs dating algorithm can be pointed out as:

- Extract TPs for each individual time series n = 1, 2, ..., N (eg with help of the Bry-Boschan procedure);
- Group peaks and troughs records of each time series in the sets $\{P_1, P_2, \dots, P_N\}$, $\{T_1, T_2, \dots, T_N\}$ respectively;
- Define variables $DP_{n,t}$ and $DT_{n,t}$ to measure the distance between actual period *t* and a nearest peak or trough
- For each *t* compute median of $DP_{n,t}$ and $DT_{n,t}$ across n = 1, 2, ..., N;
- Select local minima and maxima within a sliding window;
- Remove TPs which are too close (15 months for monthly time series) and series of consecutive peaks or troughs.

Harding-Pagan turning point detection procedure is used in this article as a naive (benchmark) business cycle dating algorithm.

4. Historical time series and real-time database

Historical time series and real-time database was set up for the time period 1990:M1–2010:M12. Historical time series were gathered for period 1990:M1–2005:M12 and vintage database embraced time span 2006:M1–2010:M12. Financial data used for realized volatility estimation were taken from DataStream database and consisted of 15-minute sampling ticks last transaction prices. The scope of the data was dependent on the country for which the MS-DFM model was applied:

- US: USD/EUR, DJIA (Dow Jones Composite Average);
- Germany: USD/EUR(ECU)/DM, DAX 30;
- France: USD/EUR(ECU), CAC 40;
- Italy: USD/EUR(ECU), MIB GENERAL;
- UK: GBP/EUR(ECU), FTSE 100.

For each country monthly macroeconomic data was also used in the estimation (sources: US Federal Reserve Board /FRED/, BEA, Eurostat, ECB Statistical Data Warehouse):

- US: index of industrial production, personal income less transfer payments, employees on nonagricultural payments;

- Germany: industrial production, retail sales, employment (number of people employed);
- France: industrial production, oersonal consumption, number of employees in the private non-agricultural sector;
- Italy: index of industrial production, private consumption, number of employees in manufacturing;
- UK: industrial production, real household disposable income, employment (LFS).

Also quarterly GDP was used as a real economy reference variable.

5. Results

The sequence of the next five figures reports probabilities of recession computed with Markov Switching Dynamic Factor Model for the surveyed countries. Recession probabilities were computed for two versions of the reference model: without (version signed as MS-DFM) and with appropriate realised volatility time series included (MS-DFM RV). Achieved results were compared with recession periods (highlighted as gray areas) dated by National Bureau of Economic Research (NBER) for US and Economic Cycle Research Institute (ECRI) for Western European countries.

As it can be seen from the first picture introduction of RV series into modeling framework significantly improved the time lead of 2001 crisis detection in the US. It was as well helpful to improve timeliness of the last crisis forecast. However it is worthy to notice that inclusion of RV series generated also a spurious US recession signal paralelly with outbreak of ‘Russian Flu’ crisis in 1998.

Realised volatility of financial variables were also helpful in improving the quality of recession forecasting in the case of Germany and France. Similarly to the US situation inclusion of highly volatile financial data generated one additional recession signal in 1998.

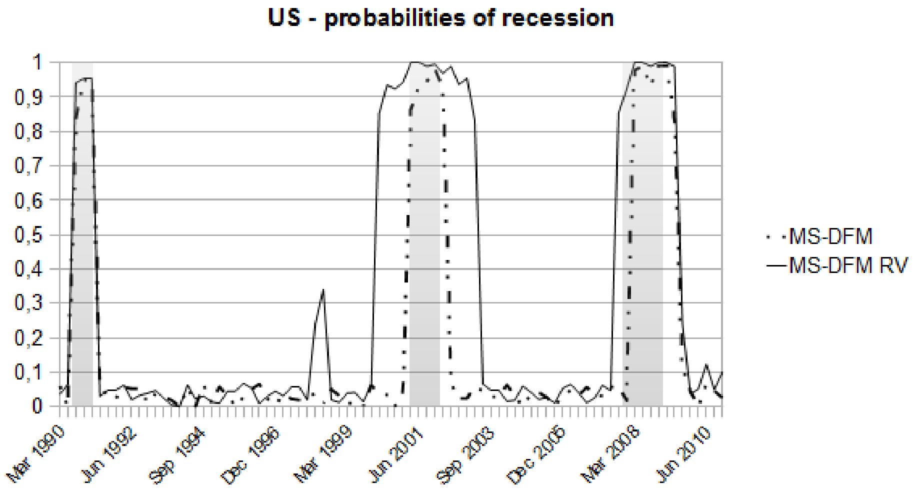

Figure 1. MS-DFM for US.

Source: own computations.

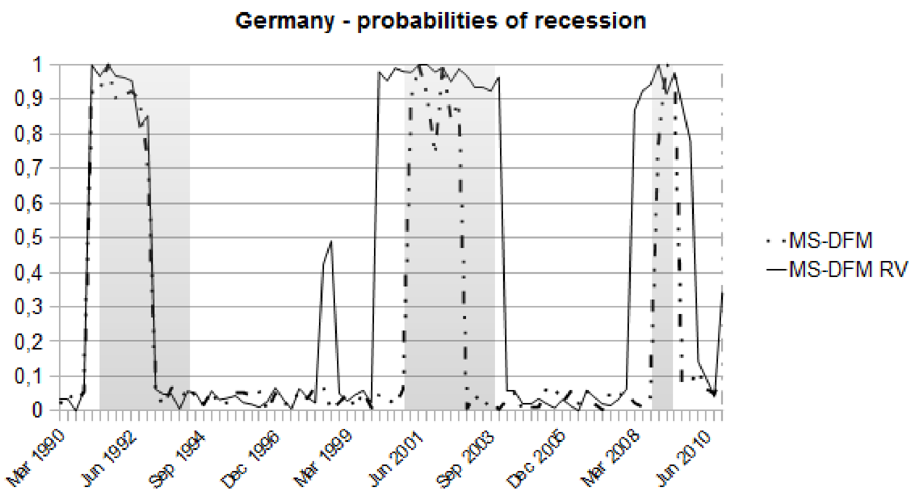

Figure 2. MS-DFM for Germany.

Source: own computations.

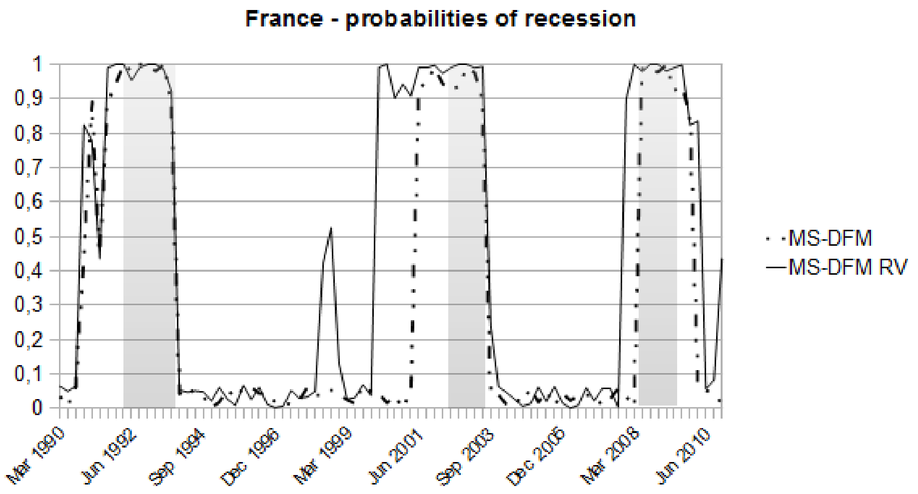

Figure 3. MS-DFM for France.

Source: own computations.

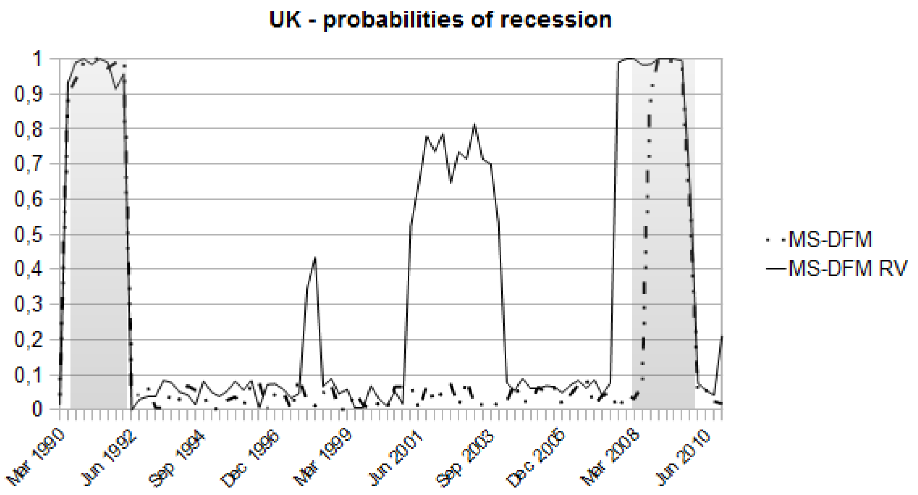

Figure 4. MS-DFM for UK.

Source: own computations.

Much worse results are observed for the two remaining countries: UK and Italy. These countries avoided recession of 2001 but inclusion of RV series caused spurious recession signal at the end of 2000. It seems that high volatility of domestic financial data caused by global financial markets turbulences was propagated into the recession forecasting model and generated false alarms.

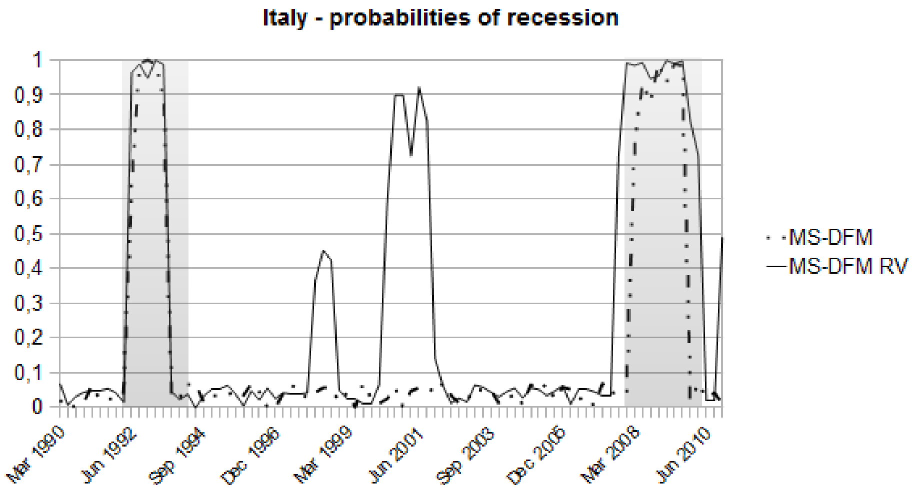

Figure 5. MS-DFM for Italy.

Source: own computations.

In the second exercise the output of MS-DFM (RV) turning point detection procedure was compared with the benchmark dating algorithm of Harding & Pagan (HP without and with RV data). For the first two countries (US and Germany) the lead time of MS-DFM TPs detection was generally the same as measured with the HP method. The MS-DFM showed its advantage over HP in the case of France giving policy makers one and four more quarters to take precautionary actions in 1992 and 2002 respectively. Introduction of RV time series changed this picture. In the case of US the application of MS-DFM with RV series lengthens the time lead for the 2001 peak to 4 quarters from 2 quarters for HP RV and to 1 quarter (from 0) for the 2007 local maximum Similar differences were noticed in the case of Germany (-4/-1 and 2001, -3/-1 for 2008 respectively) and France (-6/-10 for 2001).

Table 3. US TPs' dates

TP	MS-DFM	MS-DFM RV	HP	HP RV
P	1990:Q3	1990:Q3	1990:Q3	1990:Q3
T	1991:Q1	1991:Q1	1991:Q1	1991:Q1
P	2001:Q1	2000:Q1	2001:Q1	2000:Q3
T	2002:Q1	2003:Q1	2002:Q1	2002:Q2
P	2008:Q1	2007:Q3	2008:Q1	2007:Q4
T	2009:Q3	2009:Q4	2009:Q3	2009:Q3

Source: own computations.

Table 4. US recession dates

Peak	NBER	MS-DFM	MS-DFM RV	HP	HP RV
1	1990:Q3	0	0	0	0
2	2001:Q1	0	-4	0	-2
3	2007:Q4	1	-1	1	0

Source: own computations.

Table 5. Germany TPs' dates

TP	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
P	1991:Q1	1991:Q2	1991:Q1	1991:Q2	1991:Q2
T	1994:Q2	1992:Q4	1992:Q4	1994:Q1	1993:Q4
P	2001:Q1	2001:Q1	2000:Q1	2001:Q1	2000:Q4
T	2003:Q3	2002:Q3	2003:Q4	2003:Q2	2003:Q4
P	2008:Q4	2008:Q4	2008:Q1	2008:Q4	2008:Q3
T	2009:Q1	2009:Q2	2009:Q4	2009:Q1	2009:Q2

Source: own computations.

Table 6. Germany recession dates

Peak	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
1	1991:Q1	0	0	1	1
2	2001:Q1	0	-4	0	-1
3	2008:Q4	1	-3	0	-1

Source: own computations.

Table 7. France TPs' dates

TP	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
P	1992:Q1	1991:Q1	1990:Q4	1991:Q4	1991:Q2
T	1993:Q3	1993:Q3	1993:Q3	1993:Q3	1993:Q3
P	2002:Q3	2001:Q2	2000:Q1	2001:Q4	2001:Q1
T	2003:Q2	2003:Q2	2003:Q2	2003:Q2	2003:Q2
P	2008:Q1	2008:Q1	2007:Q4	2008:Q1	2007:Q4
T	2009:Q1	2009:Q4	2010:Q1	2009:Q2	2009:Q3

Source: own computations.

Table 8. France recession dates

Peak	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
1	1992:Q1	-4	-5	-1	-3
2	2002:Q3	-5	-10	-3	-6
3	2008:Q1	0	-1	0	-1

Source: own computations.

The gained results were not so clear in the case of UK and Italy. Although the use of the MS-DFM RV model lengthens lead of 2007/2008 recession detection for UK (to 3 from 2 quarters), it didn't allow to detect in advance the business cycle peaks in Italy.

Table 9. UK TPs' dates

TP	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
P	1990:Q2	1990:Q2	1990:Q2	1990:Q2	1990:Q2
T	1992:Q1	1992:Q1	1992:Q1	1992:Q1	1992:Q1
P	-	-	2001:Q1	-	-
T	-	-	2003:Q4	-	-
P	2008:Q2	2008:Q3	2007:Q3	2008:Q2	2007:Q4
T	2010:Q1	2009:Q4	2009:Q4	2009:Q4	2009:Q4

Source: own computations.

Table 10. UK recession dates

Peak	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
1	1990:Q2	0	0	0	0
2	2008:Q2	0	-3	0	-2

Source: own computations.

Table 11. Italy TPs' dates

TP	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
P	1992:Q1	1992:Q2	1992:Q2	1992:Q2	1992:Q2
T	1993:Q4	1993:Q2	1993:Q2	1993:Q3	1993:Q2
P	-	-	2000:Q1	-	-
T	-	-	2001:Q3	-	-
P	2007:Q3	2008:Q1	2007:Q3	2008:Q2	2007:Q4
T	2010:Q1	2009:Q3	2010:Q1	2009:Q4	2009:Q4

Source: own computations.

Table 12. Italy recession dates

Peak	ECRI	MS-DFM	MS-DFM RV	HP	HP RV
1	1992:Q1	1	1	1	1
2	2007:Q3	2	0	3	1

Source: own computations.

6. Conclusions

Taking into account the output of the applied models, the achieved results of described survey are mixed:

1. Application of stock and currency markets realised volatility time series in MS-DFMs generally helped to increase 'speed' of recession prediction, but it could be also perceived as a source of false signals when financial and real economy situation were decoupled.

2. MS-DFMs can distort the length of business cycle phases (recession phases are shorter usually than official).

3. Application of Harding & Pagan TPs dating algorithm allows to keep phases of a business cycle more consistent with official ones, however using this algorithm diminished the lead of TPs' dating.

For the future author plans to include into applied the MS-DFM model and Harding & Pagan TPs dating procedure volatilities from bond markets and examine different specifications of the Markov switching component in the DFM structure.

References

Andersen, T. G., Bollerslev, T., Cai, J. (2000). Intraday and interday volatility in the Japanese stock market. *Journal of International Financial Markets, Institutions and Money*, Elsevier, 10 (2), 107-130.

Andersen, T. G., Bollerslev, T., Diebold, F. X, Labys, P. (2001). The distribution of realized exchange rate volatility. *Journal of the American Statistical Association*, 96, 42-55.

Andersen, T. G., Bollerslev, T., Diebold, F.X., Labys, P. (2003). Modeling and forecasting realized volatility, *Econometrica*, 71 (2), 579-625.

Andersen, T. G., Bollerslev, T., Meddahi, N. (2004). Analytical evaluation of volatility forecasts. *International Economic Review*, 45 (4), 1079-1110.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Vega, C. (2007). Real-time price discovery in global stock, bond and foreign exchange markets. *Journal of International Economics*, 73 (2), 251-277.

Ang, A., Bekaert, G. (2002). International asset allocation with regime shifts. *Review of Financial Studies*, 15 (4), 1137-1187.

Barndorff-Nielsen, O. E. (2004). Power and bipower variation with stochastic volatility and jumps, *Journal of Financial Econometrics*, 2 (1), 1-37.

Barndorff-Nielsen, O. E., Graversen, S. E., Jacod, J., Shephard, G. (2005). Limit theorems for bipower variation in financial econometrics. *OFRC Working Papers Series*.

Camacho, M., Pérez-Quirós, G. (2009). Introducing the Euro-STING: Short-term indicator of Euro Area growth. *CEPR Discussion Papers*, 7343.

Chauvet, M. (1998). An econometric characterization of business cycle dynamics with factor structure and regime switching. *International Economic Review*, 39 (4), 969-96.

Chauvet, M., Hamilton, J. D. (2005). Dating business cycle turning points. *NBER Working Papers*, 11422, National Bureau of Economic Research.

Chauvet, M., Piger, J. (2008). A comparison of the real-time performance of business cycle dating methods. *Journal of Business & Economic Statistics*, 26, 42-49.

Corsi, F., Mittnik, S., Pigorsch, C., Pigorsch, U. (2008). The volatility of realized volatility. *Econometric Reviews*, 27 (1-3), 1-33.

Garcia, R., Perron, P. (1996). Na analysis of the real interest rate under regime shifts. *The Review of Economics and Statistics*, 78 (1), 111-125.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57 (2), 357-84.

Hamilton, J. D. (1988). A neoclassical model of unemployment and the business cycle. *Journal of Political Economy*, 96 (3), 593-617.

Harding, D., Pagan, A. (2002). Dissecting the cycle: a methodological investigation. *Journal of Monetary Economics,* 49 (2), 365-381.

Kim, Ch. J. (1994). Dynamic linear models with Markov-switching. *Journal of Econometrics*, 60 (1-2), 1-22.

Kim, Ch. J., Nelson, Ch. (1999). *State-space Models with Regime Switching*. Cambridge: MIT Press.

Mariano, R. S, Murasawa, Y. (2003). A new coincident index of business cycles based on monthly and quarterly series. *Journal of Applied Econometrics*, 18 (4), 427-443.

Stock, J. H., Watson, M. W. (1989). New indexes of coincident and leading economic indicators. *NBER Macroeconomics Annual*, 4, 351-409.

Stock, J. H., Watson, M. W. (1998). Business cycle fluctuations in U.S. macroeconomic time series. *NBER Working Papers*, 6528.