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The Viterbi paths in an analysis of business cycle synchronization

Abstract

In the paper we investigate possibility of using the Viterbi paths to analyze two-dimensional macroeconomic time series. We build a two-dimensional Gaussian Markov-switching model with a four-state hidden Markov chain. The model is tested with two pairs of monthly indexes of industrial production for: Poland vs. France, and Poland vs. Germany. The most likely sequence of states of the hidden Markov chain is found for each pair. We compare that sequence with analogous sequences determined for a one-dimensional model with a two-state hidden Markov chain. The results of the comparison suggests the four-state Viterbi path provides more valuable information about business cycle synchronization between the two economies than two separate two-state Viterbi paths.

Keywords: Markov switching models, Viterbi path, business cycle synchronization

JEL classification: C63, E37, E27

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1. Introduction

The paper proposes the so-called Viterbi paths to analyze business cycle synchronization. The Viterbi path is the most probable sequence of a hidden Markov chain in a Markov switching model (MS model). It is widely used for speech recognition and DNA analysis, but almost absent in econometrics despite its great usefulness in non-linear modeling. The advantage of the proposed method is, among others, simple interpretation of results. This is important especially when a hidden Markov chain has more than two states. The usefulness of the method for a business cycle analysis has been confirmed empirically in case of univariate time series (Bernardelli & Dędys, 2012). In this paper we show that the Viterbi path can also be a valuable tool when analyzing bivariate time series. We consider two time series of the sold industrial production index for two economies: 'weaker' and 'stronger'. The states of the unobservable Markov chain reflect changes in business climate in both economies. For this purpose we use a model with four hidden states.

This paper is organized as follows. In the second section the brief history and literature overview is done. In the third section the theory of switching Markov models, as well as the description of the data used in the study are presented. The results of the empirical analysis are given in the fourth section. The paper ends with conclusions.

2. Markov switching models and the synchronization of the business cycles

The application of Markov switching models to analyze business cycles has been long present in econometric modeling (Hamilton, 1989). These models are mainly used to determine the rates of growth or to detect business cycle turning points. A variety of types of such models is impressive and concerns both an observable and an unobservable component. It is often assumed that an observable component, roughly speaking, is generated by autoregressive (AR) or vector autoregressive (VAR) processes with time-varying parameters (in these cases we say about MS-AR and MS-VAR, respectively).

The unobservable component can be either a homogeneous or heterogeneous Markov chain. In the heterogeneous case logistic functions of some exogenous variables are assumed to be transition probabilities. Such models are called the Markov switching models with time-varying transition probabilities (MS-TVTP) (Moolman, 2004; Simpson *et al.*, 2001). More importantly, in vast majority of models two hidden states are considered, where the transition probabilities change in time or not. In a natural way these states correspond to two phases of a business cycle. However, in some studies three states are considered. This aims to distinguish a recession, post-recession rapid recovery and moderate growth (Boldin, 1996), or recession, normal growth and high-growth episodes (Artis *et al*., 2004).

As mentioned, the turning points detection is one of the most important applications of MS models in a business cycle analysis. In general, the identification of different phases of a business cycle is provided by estimates of filtered or smoothed probabilities (Chauvet & Hamilton, 2005). These probabilities could also give a basis for an analysis of business cycle synchronization of different economies. One way to make such a comparison is fitting MS-AR models to data for each country separately, estimating the mentioned probabilities and inference using appropriate measures (Smith & Summers, 2005). In some research, which is focused on determining a common business cycle, MS-VAR models are applied. An interesting alternative is provided by MS-TVTP models of the AR type. In these models the observable component is associated with a business cycle of one economy, while the potential relation to business fluctuations of another economy is reflected in the unobservable component. More specifically, the transition probabilities of the hidden Markov chain in such a model are functions of the latter (Dufrénot & Keddad, 2014). Another approach to an analysis of business cycle synchronization for a pair of economies is to consider the bivariate observable component and the hidden Markov chain with four states reflecting economic climate in both economies (Phillips, 1991). In our study we use that approach.

3. Model description and data characteristics

In this paper we focus on the simplest kind of MS model. Namely, we deal with conditionally independent observable variables with parameters of distribution that are driven by a homogenous Markov chain. More precisely, we consider partially observable process $\{(X_t, Y_t)\}_{t=1}^{\infty}$ satisfying the following condition:

1. Unobservable component $\{X_t\}_{t=1}^{\infty}$ is a homogenous Markov chain with finite state space S_X.
2. Observable random variables Y_1, Y_2, …, Y_t given $(X_1, X_2, \dots, X_t)$ are conditionally independent, and distribution of Y_t given this condition depends only on the random variable X_t.

Markov chain $\{X_t\}_{t=1}^{\infty}$ is called the hidden Markov chain. Models of this type are known as hidden Markov models (HMM), and appeared in the literature in the 1960s, that is much earlier than the first work of Hamilton (Cappé *et al*., 2005).

One of the major issues in the application of MS is as follows. Having information about the realization of observable variables Y_t in some period of time (say from 1 to *T*), one could try to estimate the state of the unobservable MC at fixed time t, where $t \leq T$. The most common approach is to use the smoothed probability:

$$w_t(i) = P(X_t = i | Y_1 = y_1, Y_2 = y_2, .., Y_T = y_T) \tag{1}$$

or the filtered probability:

$$f_t(i) = P(X_t = i | Y_1 = y_1, Y_2 = y_2, .., Y_n = y_n) \tag{2}$$

to deal with this problem.

There are several procedures for obtaining the assessment of the state of the hidden Markov chain at time *t*, which use estimates of filtered or smoothed probabilities (Chauvet & Hamilton, 2005; Harding & Pagan, 2002). In the simplest case $\underset{i}{\operatorname{argmax}} w_t(i)$ or $\underset{i}{\operatorname{argmax}} f_t(i)$ gives this assessment. Unfortunately, such 'local decoding' or 'step-by-step decoding- of the path of the states of the hidden Markov chain may be ineffective, especially in the case of larger state space. In this paper we use an alternative method to solve this problem. Specifically, we look for the most likely path of MC in the whole period covered in the analysis. Formally speaking, we determine the path $(x_1^*, x_2^*, \dots, x_T^*) \in S_X^T$ such that:

$$P(X_1 = x_1^*, \dots, X_T = x_T^* | Y_1 = y_1, .., Y_T = y_T) = \\ \max_{(x_1, x_2, \dots, x_T) \in S_X^T} \{P(X_1 = x_1, \dots, X_T = x_T | Y_1 = y_1, \dots, Y_T = y_T)\}. \tag{3}$$

This most likely sequence is called the Viterbi path after Andrew Viterbi, the author of the algorithm used to determine the path.

For the estimation of the parameters of hidden Markov models the iterative Baum-Welch algorithm are used (Cappé *et al*., 2005). However, results of this deterministic algorithm depend on initial values of the probabilities. Therefore, they may be far from optimal. In order to increase chances of finding the optimal solution, the calculation can be repeated several times for the same set of data and different initial values. This is equivalent to performing a Monte Carlo simulation. For each *k*-state HMM model preselecting of the following values is required:

- initial distribution of an unobserved Markov chain (*k* parameters),
- transition probabilities of unobserved Markov chain parameters

(k^2 parameters),
- means and covariances of the conditional distribution of an observed variable in the given state ($2k$ parameters).

In this research the initial values were chosen randomly using independent and identically distributed draws from the univariate distribution. The number of draws used for the parameters estimation of the time series under the study varied between 1000 and 5000. The number of draws depend on a number of HMM's states and the numerical stability of computations.

The best estimates of parameters of models were chosen with selection criteria which take into account the following indicators (Bernardelli, 2015; Bernardelli & Dędys, 2014):
- Akaike's information criterion (AIC),
- Bayesian information criterion (BIC),
- the log likelihood value,
- frequency of obtaining certain solution of the Baum-Welch algorithm (with an accuracy of one decimal place).

The HMM model, considered as the best for the particular input data set, was used to compute the most likely path which consists of the sequence of states of MC (throughout the whole period under consideration). These paths are outputs of the Viterbi algorithm (Cappé *et al*., 2005). It is worth noting that despite of the deterministic nature of both algorithms, the method of 'decoding' states of unobserved MC as a whole has a non-deterministic character.

In this paper we consider MS with observable variable Y_t having univariate or bivariate Gaussian conditional distribution and two or four hidden states. MS model with two unobservable states we use for modeling univariate time series. In this case $S_X = \{0,1\}$ and:

$$Y_t|_{X_t=0} \sim N(\mu_0, \sigma_0), \quad Y_t|_{X_t=1} \sim N(\mu_1, \sigma_1), \tag{4}$$

where $\mu_0 < \mu_1$. State 0 corresponds to the periods of contraction, and state 1 relates to the periods of expansion.

In order to capture possible interactions between pairs of economies we introduce a model similar to the model proposed in (Phillips, 1991). We focus on bivariate time series with components corresponding to individual economies under the study. By considering the hidden Markov chain with state space $S_X = \{(0,0), (0,1), (1,0), (1,1)\}$, we expect that the state (0,0) should relate to the periods in which both economies are in the contraction phase. The interpretation of other states should be analogous. In contrast to the model presented in (Phillips, 1991) we consider the simplest MS model with conditional Gaussian distributions. We put no restrictions on the vectors of expected values of these distributions. In addition, conclusions about the possible interactions between economies are drawn on the basis of the Viterbi paths, and not on the basis of the matrix of transition probabilities.

In this paper we analyze the monthly index of industrial production[1] (IIP) of Germany, France and Poland. The data are taken[2] from the Eurostat database, and the research covers the period from January 2002 to January 2015. The percentage change compared to the same month of the previous year was chosen as the unit. It helps to avoid necessity of seasonal adjustment and possible problems that can be caused by such data transformation (Matas-Mir A. *et al*., 2008).

4. Results of empirical analysis

It is worth emphasizing that the Gaussian MS model and Viterbi paths could not be treated as a universal tool for an analysis of time series. To examine the effectiveness of the proposed method on data under the study, some comparisons are made. First, we confront the Viterbi paths obtained for the hidden MC with two states with the business cycle turning points dated by OECD on the basis of Composite Leading Index (Figures 1-3).

Unfortunately, in the case of Germany (Figure 2) and France (Figure 3) the strong effects of the financial crisis in 2008 disrupted the decomposition. Therefore, the decision was made to perform some 'local smoothing' of the time series data. This was done by a proportional increase of the value of IIP in relation to other crises in the years preceding 2008. The results of the procedure are given in Figure 4 (Germany) and Figure 5 (France). Transformed in such a way, time series were taken for further analysis.

[1] More exactly, a percentage change compared to the same period in the previous year (data adjusted by working days, covering mining and quarrying, manufacturing, electricity, gas, steam, air conditioning supply, and construction).

[2] Accessed 23 November 2015.

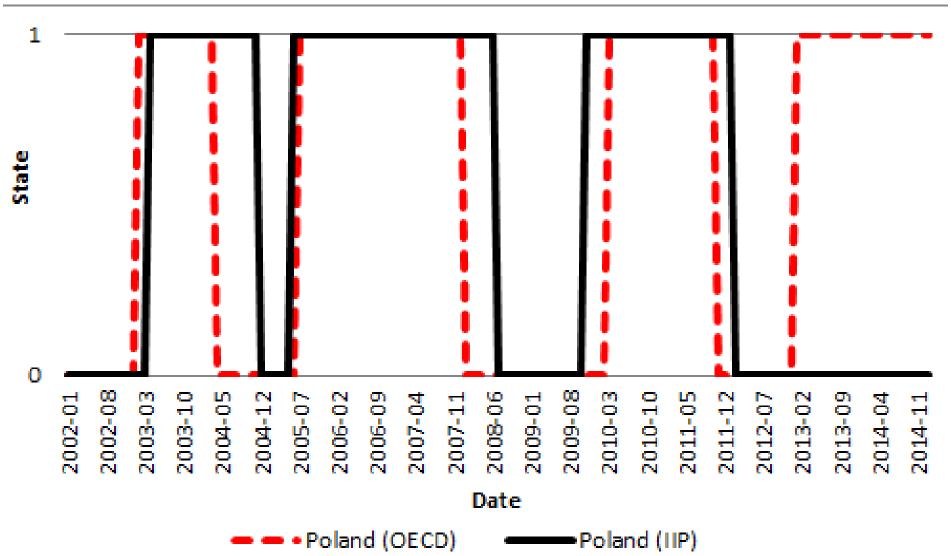

Figure 1. Comparison of the Viterbi path of the two state HMM for the indexes of industrial production in Poland with the OECD reference time series.

Source: own calculations.

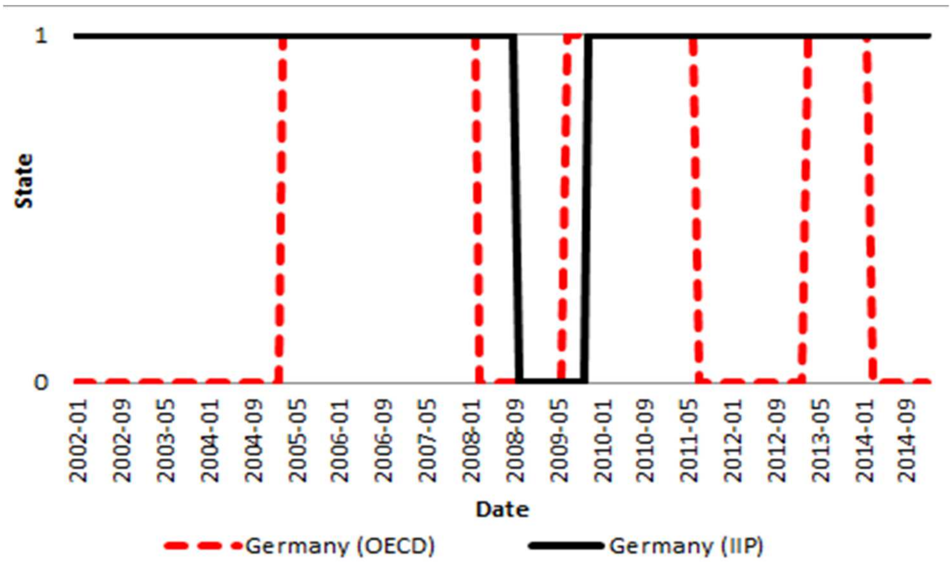

Figure 2. Comparison of the Viterbi path of the two state HMM for the indexes of industrial production in Germany with the OECD reference time series.

Source: own calculations.

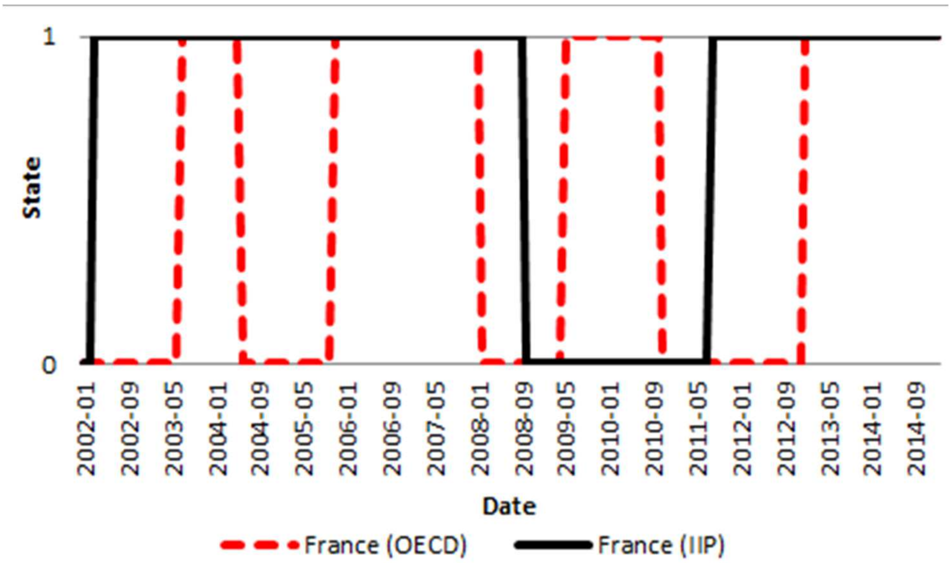

Figure 3. Comparison of the Viterbi path of the two state HMM for the indexes of industrial production in France with the OECD reference time series.

Source: own calculations.

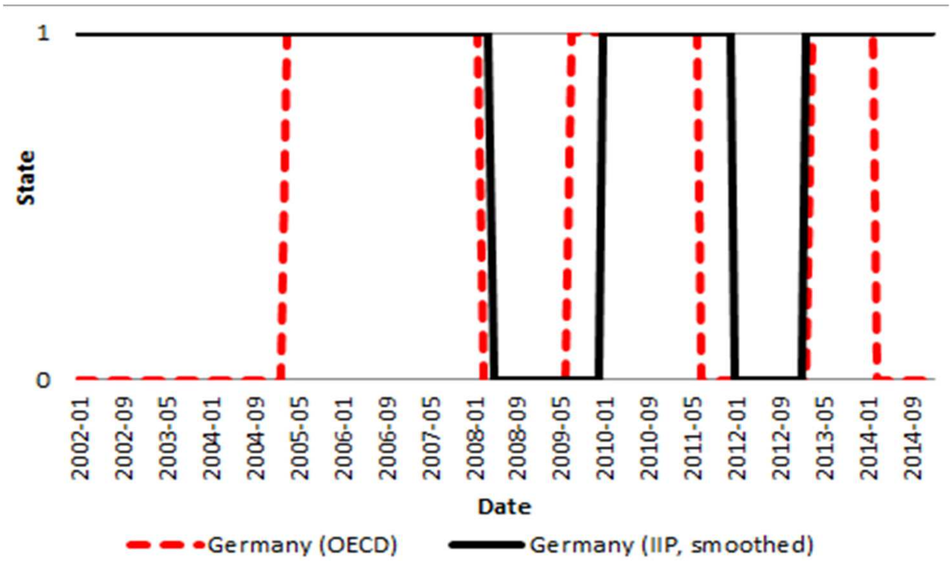

Figure 4. Comparison of the Viterbi path of the two state HMM for the locally smoothed indexes of industrial production in Germany with the OECD reference time series.

Source: own calculations.

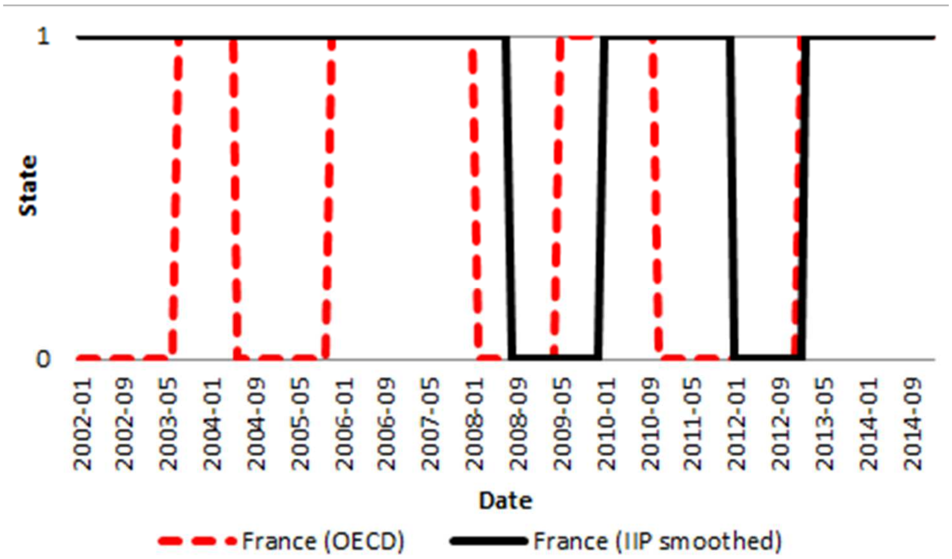

Figure 5. Comparison of the Viterbi path of the two state HMM for the locally smoothed indexes of industrial production in France with the OECD reference time series.
Source: own calculations.

The results show that almost all turning points dated by the OECD are reflected in the Viterbi paths. One turning point is missed in the path of Poland, and two are missed in the case of Germany. And all turning points before 2008 are not present in the Viterbi path of Poland. In general, signals about turning points appear in the Viterbi paths with some delay. These lags are equal to 2-8 months (Poland), 1-6 months (Germany) or 1-14 months (France). Only in the case of Poland and Germany some turning points are signaled by the Viterbi path in advance.

In this paper the business cycle analysis is carried out only on the basis of Viterbi paths. In order to examine an impact of the stronger economy on the weaker economy, another kind of comparison was made. Based on the Viterbi path for one-dimensional series of IIP and two state hidden Markov chains, we constructed ‘compound paths’ as follows. States 0 and 1 were assigned in the case of consistent indications: if in both considered paths the situation was identified as contraction, then in the constructed compound path the state was taken as 0. The procedure was analogous for the state 1: if there is an agreement in each of two paths about expansion in a particular period, then in the compound path the state 1 was assumed. State ⅔ was introduced to reflect an expansion phase of the weaker economy and the contraction phase of the stronger economy. State ⅓ was established to describe the opposite. The compound paths may give information from the business cycles considered separately. These compound paths against the background of paths determined on the basis of univariate models for the weaker economy are shown in Figures 6 (Poland vs. Germany) and 7 (Poland vs. France).

The main goal of the study was to explore applicability of the bivariate HMM case to an analysis of business cycles synchronization. Therefore, for the decomposition of time series of pairs of considered countries we used MS model with four hidden states, which was introduced in the previous section. For the clarity of the visualization of the Viterbi paths, we renumbered the states of the hidden Markov chain. State (0,0) is denoted by 0, and state (1,1) by 1. States (0,1) and (1,0) correspond to middle states ⅓ and ⅔. However the results of estimation, especially for means of Gaussian distribution, suggest that interpretation of these middle states should be slightly different compared to the original assumptions. In Figure 8 (Poland and Germany) and Figure 9 (Poland and France) the Viterbi paths obtained for bivariate MS models are shown against the weaker economies, while the stronger economies are presented as a background in the Figure 10 (Germany) and Figure 11 (France).

At least two comparisons should be discussed. The first is the confrontation of the results from models with two states and HMM with four states (both compound paths and the Viterbi paths of bivariate models). It seems that the Viterbi path connected with 4-state models enrich inference about business cycle turning points. Two middle states may signal the turning points. What is more, those signals seem to indicate clearly the direction of the change when the strongly linked economies are considered (Germany and Poland). It accounts for the usefulness of the models with states extended to more than two.

The resulting paths from the bivariate models over the compound paths are advantageous. They are definitely smoother and usually give signals earlier than the analogous announcement in the corresponding compound path. They simply better reflect changes in the economy.

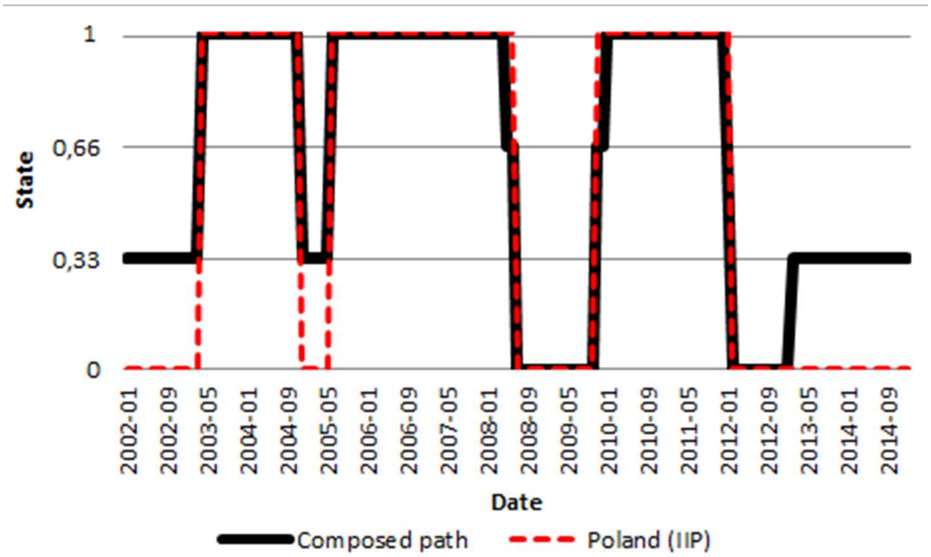

Figure 6. Comparison of the compound Viterbi path calculated based on the two state HMMs for the indexes of industrial production in Poland and Germany with the Viterbi path of the two state HMM for IIP of Poland.
Source: own calculations.

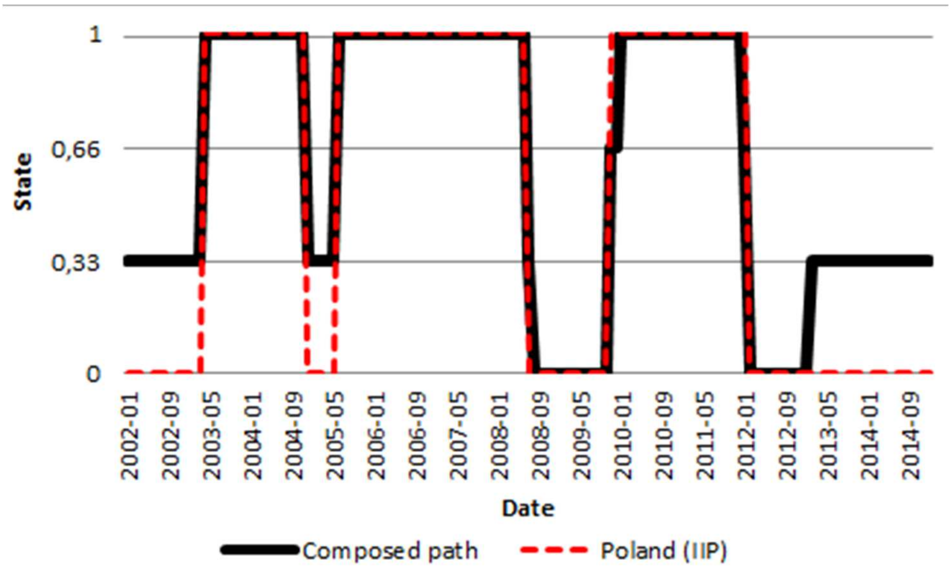

Figure 7. Comparison of the compound Viterbi path calculated based on the two state HMMs for the indexes of industrial production in Poland and France with the Viterbi path of the two state HMM for IIP of Poland.
Source: own calculations.

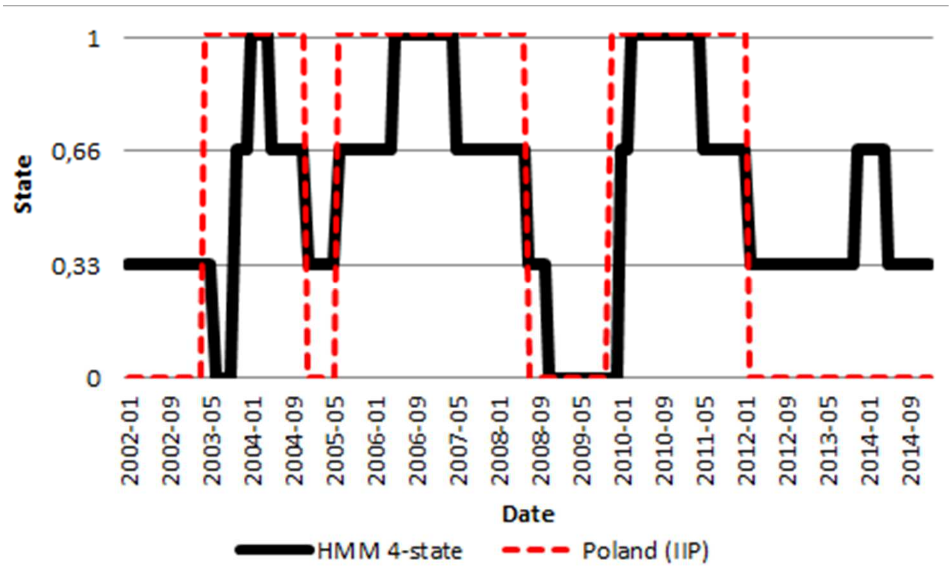

Figure 8. Comparison of the Viterbi path of the four state HMM for the indexes of industrial production in Poland and Germany with the Viterbi path of the two state HMM for IIP of Poland.
Source: own calculations.

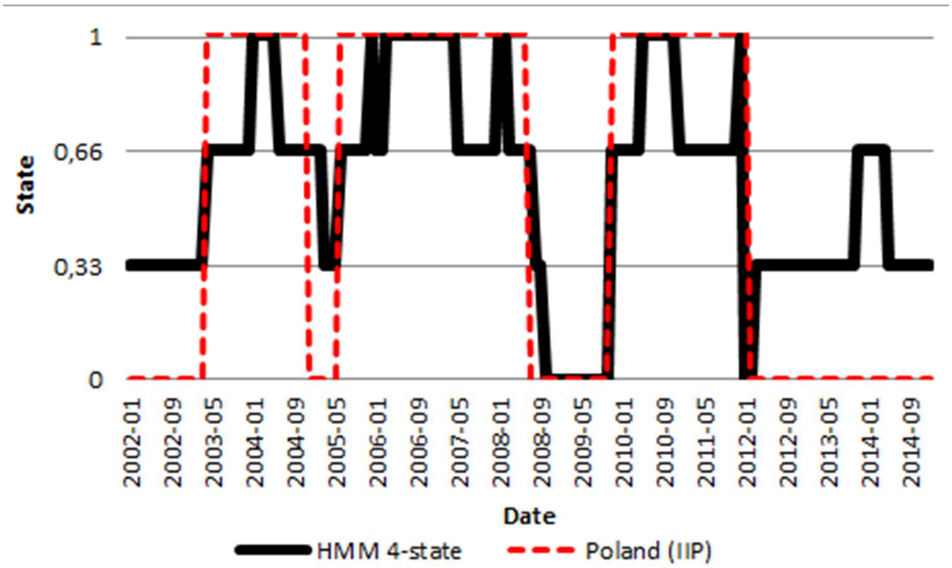

Figure 9. Comparison of the Viterbi path of the four state HMM for the indexes of industrial production in Poland and France with the Viterbi path of the two state HMM for IIP of Poland.
Source: own calculations.

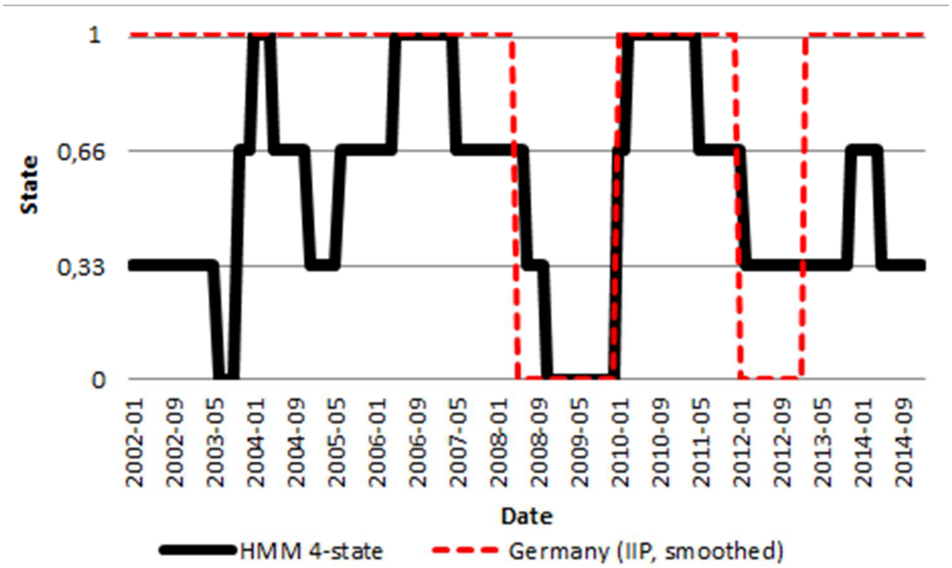

Figure 10. Comparison of the Viterbi path of the four state HMM for the indexes of industrial production in Poland and Germany with the Viterbi path of the two state HMM for IIP of Germany.
Source: own calculations.

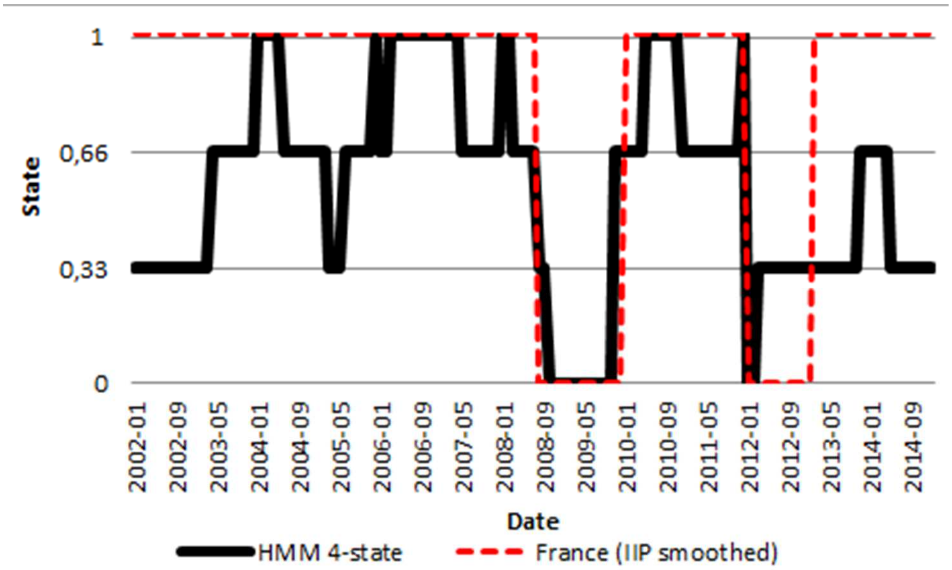

Figure 11. Comparison of the Viterbi path of the four state HMM for the indexes of industrial production in Poland and France with the Viterbi path of the two state HMM for IIP of France.
Source: own calculations.

5. Conclusions

The results of the study suggest that the Viterbi paths can be a valuable tool for an analysis of bivariate time series whose components correspond to macroeconomic time series of two interconnected economies. The method was found useful for studying industrial production of the following pairs of economies: Germany and Poland, and France and Poland. Two-dimensional Gaussian Markov switching models with four-state hidden Markov chains were used in the analysis. The Viterbi paths generated by these models give more valuable information in comparison to the paths provided by the separated models with univariate observable components.

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