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Mapping the respondents' assessments in business tendency survey using the Viterbi paths

Abstract

In the paper we propose to use the so-called Viterbi paths for mapping relationships between survey data. The Viterbi path is the most probable sequence of states of a hidden Markov chain in a Markov Switching model (MS). The approach is widely taken to recognize speech or to analyze DNA, but is almost absent in econometrics, despite the great role MS models play in non-linear modeling. The main advantages of the Viterbi paths are: (1) intuitive interpretation of results they give and (2) their wide applicability. They have, however, some disadvantages too. It turns out that the models we have built do not necessarily fit to business tendency survey data, and the interpretation of the hidden states might be unclear.

Keywords: business tendency surveys, wording of survey questions.

JEL: C81, E32.

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1. Introduction

The Viterbi paths are strongly connected with Markov Switching models and problems of pattern recognition. They can be regarded as a powerful tool for comparison of univariate and multivariate macroeconomic time series. Their usefulness has been shown in detecting turning points, or identifying periods of a certain level or a change rate of economic phenomena under consideration (Bernardelli, Dędys, 2014).

The goal of this paper is to show a variety of applications of the Viterbi paths. We study the data that comes from the business tendency survey the Research Institute for Economic Development, Warsaw School of Economics (RIED) conducts in the Polish manufacturing industry every month. Specifically, we analyze time series of the state balances of: production, total orders, finished goods inventories, selling prices, employment and financial standing of manufacturers. Any detailed description of business situation in the industry, reported by respondents of the survey, is beyond the scope of the paper. We mainly focus on presenting how the Viterbi paths work.

The Viterbi paths, obtained from the univariate time series of the balances, answer the following questions: (1) Is it possible to separate sets of time series which are strongly synchronized? (2) Is there any leading one out of the balances? (3) Are there any 'local leaders', that is, the time series which lead downturns or upturns only? and, finally, (4) Is there any recommendation to apply the two-, three- or four-states Viterbi paths?

The Viterbi paths can be a valuable tool to analyze bivariate time series as well. As Bernardelli and Dędys (2015) show, they could be applied to evaluate business cycle synchronization, especially when 'weaker' vs 'stronger' economies are considered. With this in mind, we focus on the two following problems: (1) Are the changes, that take place in an economy, noticed by manufacturers earlier than the corresponding changes in their firms, or *vice versa*? (2) What is the order of signaling when the balances related to leading and coincident indicators are taken into account? Unfortunately, in the two-dimensional analysis some difficulties may appear, i.e., unlike the one-dimensional analysis, a universal interpretation of the states in the Viterbi path is not plausible. Furthermore, it is not always possible to estimate a model for selected pairs of the balances. We briefly discuss these problems.

The paper is organized as follows. Section 2 is devoted to brief discussion of the basic terminology and methodology. The results of the empirical analysis are presented in Section 3. The paper conclude with a summary of the key findings in Section 4.

2. Markov Switching models and the Viterbi path

In this paper, we focus on the simplest type of a Markov Switching model (MS). Namely, we analyze conditionally independent observable variables with parameters of distribution driven by a homogeneous Markov chain (MC). More precisely, we consider a partially observable process $\{(X_t, Y_t)\}_{t=1}^{\infty}$, satisfying the following conditions:

1. Unobservable component $\{X_t\}_{t=1}^{\infty}$ is a homogenous Markov chain with a finite state space S_X.
2. Observable random variables $Y_1, Y_2, \ldots, Y_t$, given $(X_1, X_2, \ldots, X_t)$, are conditionally independent, and the distribution of Y_t, given this condition, depends only on a random variable X_t.

The Markov chain of that type of MS models is called the hidden Markov chain. The models of the type are known as hidden Markov models, and appeared in the literature in the 1960s, i.e., much earlier than the first work of Hamilton (Cappé *et al.*, 2005).

One of the major issues involved in the application of MS is as follows. Having information about the realization of observable variables Y_t in some period of time (say from 1 to T), one could try to estimate a state of unobservable MC at a fixed time t, $t \leq T$. The most common approach is to use the smoothed probability

$$w_t(i) = P(X_t = i | Y_1 = y_1, Y_2 = y_2, \ldots, Y_T = y_T), \tag{1}$$

or the filtered probability

$$f_t(i) = P(X_t = i | Y_1 = y_1, Y_2 = y_2, \ldots, Y_t = y_t), \tag{2}$$

to deal with this problem.

There are several procedures for obtaining the assessment of states of the hidden Markov chain in time *t*, which use estimates of the filtered or smoothed probabilities (Chauvet, Hamilton, 2005; Harding, Pagan, 2002). In the simplest case $\operatorname*{argmax}_i w_t(i)$ or $\operatorname*{argmax}_i f_t(i)$ give this assessment. Unfortunately, such 'local decoding' or 'step-by-step decoding' of the path of states of the hidden Markov chain may be ineffective, especially in the case of a larger state space.

In this paper, we use an alternative method to solve that problem. Namely, we are looking for the most likely path of MC in the whole period under the study. Formally speaking, we determine the path $(x_1^*, x_2^*, \ldots, x_T^*) \in S_X^T$ such that

$$P(X_1 = x_1^*, X_2 = x_2^*, \dots, X_T = x_T^* | Y_1 = y_1, Y_2 = y_2, \dots, Y_T = y_T) = \max_{(x_1, x_2, \dots, x_T) \in S_X^T} \{P(X_1 = x_1, X_2 = x_2, \dots, X_T = x_T | Y_1 = y_1, Y_2 = y_2, \dots, Y_T = y_T\}. \quad (3)$$

This sequence, which is more likely, is called the Viterbi path[9]. The Viterbi paths concept seems to be rarely applied in an analysis of economic data, and appears to be limited to the two-state models only (Boldin, 1994).

In this paper, we consider MS with an observable variable Y_t having univariate or bivariate Gaussian conditional distribution and two, three or four hidden states. To be clear, referring to a particular type of such a model, we use the symbol MS(*k*, *n*), where *k* is the dimension of observable time series (*k* = 1, 2) and *n* is a number of states of underlying MC (*n* = 2, 3, 4).

In the case of MS(1,2) we consider $S_X = \{0, 1\}$ and

$$Y_t|_{X_t=0} \sim N(\mu_0, \sigma_0), \quad Y_t|_{X_t=1} \sim N(\mu_1, \sigma_1), \quad (4)$$

where $\mu_0 < \mu_1$. Obviously, the state 0 corresponds to periods of the lower level, and the state 1 relates to the higher level (of the variable under the study).

The analysis can be enhanced by introducing one more state, which corresponds to unclear situation, a transition from the *poor* to the *good* state of the economy, or *vice versa*, a kind of the announcement of changes. For this purpose, we introduce a Markov chain with an extended state space $S = \left\{0, \frac{1}{2}, 1\right\}$. The state $\frac{1}{2}$ shall correspond to such an uncertain, transient period. The meaning of the states 0 and 1 is the same as in the standard two state model. An extended three state model is defined as follow

$$Y_t|_{X_t=i} \sim N(\mu_i, \sigma_i), \quad (5)$$

for $i = 0, \frac{1}{2}, 1$, where $\mu_0 < \mu_{\frac{1}{2}} < \mu_1$. Additionally, we assume that $p(0,1) = p(1,0) = 0$ to reflect smoothing of changes. As said, this model is denoted by MS(1, 3).

In order to carry out a more precise classification, another model is taken into account. To distinguish *definitely good* periods, *worse but still positive*, *definitely bad* and *moderately bad* ones, we introduce the four-level

[9] After Andrew Viterbi who was the author of the algorithm used to determine this path.

scale. The assessments are associated, respectively, with the states 1, $\frac{2}{3}$, 0 and $\frac{1}{3}$ of MC. Therefore, the MS model is introduced as follows

$$Y_t|_{X_t=i} \sim N(\mu_i, \sigma_i), \tag{6}$$

for $i = 0, \frac{1}{3}, \frac{2}{3}, 1$, where $\mu_0 < \mu_{\frac{1}{3}} < \mu_{\frac{2}{3}} < \mu_1$. As previously, we assume that only transitions between adjacent states are possible, so

$$p(0,1) = p(1,0) = p\left(0, \frac{2}{3}\right) = p\left(\frac{2}{3}, 0\right) = p\left(\frac{1}{3}, 1\right) = p\left(1, \frac{1}{3}\right) = 0.$$

In the two-dimensional case, it is assumed that the hidden Markov chain reflects some common factor, which 'governs' the pairs of observable time series. In this case, the model MS(2,2), MS(2,3) and MS(2,4) are considered. For the model MS(2,*k*), we set the same state space S_X of the hidden Markov chain as for the model MS(1,*k*), *k* =2,3,4. Obviously, we also have

$$(Y_t^1, Y_t^2)|_{X_t=i} \sim N\left(\begin{bmatrix} \mu_i^1 \\ \mu_i^2 \end{bmatrix}, \Sigma_i\right), \tag{7}$$

for $i \in S_X$.

Usually, without any insight into estimates of μ_i^1 and μ_i^2, it is not possible to give a proper interpretation of the states of hidden Markov chains. Let's consider for example MS(2,2). If each of the two observable component states 0 and 1 corresponds to the lower and higher levels, respectively, there is no basis for concluding that the hidden states of MS(2,2) correspond to (1,1) and (0,0). Furthermore, it turns out that even in the case of MS(2,4) the intuitive interpretation: (0,0), (1,1), (0,1) and (1,0) can be far from reality.

To estimate parameters of hidden Markov models we use the Baum-Welch algorithm (Cappé *et al*., 2005). However, results of this deterministic algorithm depend on initial values of probabilities. They, therefore, may be far from optimal. In order to increase the chances of finding the optimal solution, the calculation can be repeated many times for the same set of data and different initial values. This is equivalent to performing a Monte Carlo simulation. For each of a *k*-state HMM model preselecting of the following values is required:

- initial distribution of an unobserved Markov chain (*k* parameters),
- transition probabilities of unobserved Markov chain parameters (k^2 parameters),

- means and covariances of the conditional distribution of an observed variable in the given state (2*k* parameters).

In our research the initial values were randomly chosen using independent and identically distributed draws from the univariate distribution. The number of draws used for parameters estimation of the time series being under the study varied between 1.000 and 5.000. The number of trial's repetitions depends on the number of the MS's states and the numerical stability of computations.

The best estimates of parameters of the models were chosen with selection criteria taking into account the following indicators (Bernardelli, 2014; Bernardelli, Dędys, 2014):

- Akaike's information criterion (AIC),
- Bayesian information criterion (BIC),
- the log likelihood value,
- the frequency of obtaining a certain solution of the Baum-Welch algorithm (with an accuracy of one decimal place).

The MS model, considered as the best for the particular input data set, was used to compute the most likely path, consists of the sequence of the states of MC (throughout the whole period under consideration). These paths are outputs of the Viterbi algorithm (Cappé *et al*., 2005). It is worth noting that, despite the deterministic nature of both used algorithms, the method of 'decoding' the states of unobserved MC as a whole has a non-deterministic character. The stability of the results of the empirical analysis was verified with the procedure presented by Bernardelli (2015), and all the Viterbi paths were found to be stable.

3. Results of empirical analysis

This paper applies models and techniques described in the previous section to the results of the business tendency survey conducted monthly by the Research Institute for Economic Development, Warsaw School of Economics in the Polish manufacturing industry. In this survey respondents evaluate current and future (expected) changes in certain areas of economic activity. The survey basically consists of eight questions. For every question there are three possible reply options: increase, decrease or no change. For each question the balance is calculated as a difference between percentages of positive and negative answers. In the study we analyze the following balances:

- volume of production (*prod),*
- volumen of total orders (*order*),
- finished good inventories (*stock*),

- selling prices of products (*price*),
- level of employment (*employ*),
- financial standing (*fin*).

The data sample covers the period from May 2004 to February 2016. All the time series were seasonally adjusted using *Seasonal* package in R, i.e., the R-interface to X-13 ARIMA-SEATS, seasonal adjustment software developed by the United Census Bureau.

3.1. Results of decomposition of univariate time series

The parameters of MS(1,2), MS(1,3) and MS(1,4) were estimated for the six balances, and the Viterbi paths for all models were obtained. To give an example, in Figure 1 the time series *prod* and its decomposition into two-, three- and four-states Viterbi path are shown, and the results of the decomposition of univariate time series of all balances are shown in Figure 2.

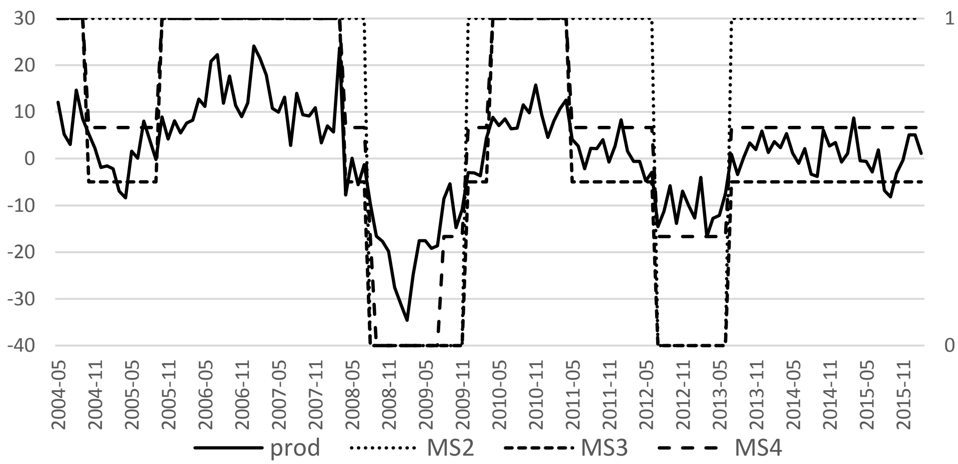

Figure 1. Original time series of the *prod* balance vs the Viterbi paths of the corresponding two-, three- and four-states MS.

We start with an analysis of the Viterbi paths obtained for MS(1,2). At first glance, one can see three very similar Viterbi paths for *prod, order* and *fin*. The two last ones are almost identical (see Figure 3). As a matter of fact, except for the first additional series of 'zeros', the Viterbi path for *employ* may be attached to that group. By analyzing Figure 4 one can see the analogous relationship in the observable time series. The Viterbi paths of *price* and *stock* differ from the rest of the paths and from each other. In addition, the Viterbi path obtained for *stock* has the greatest variability.

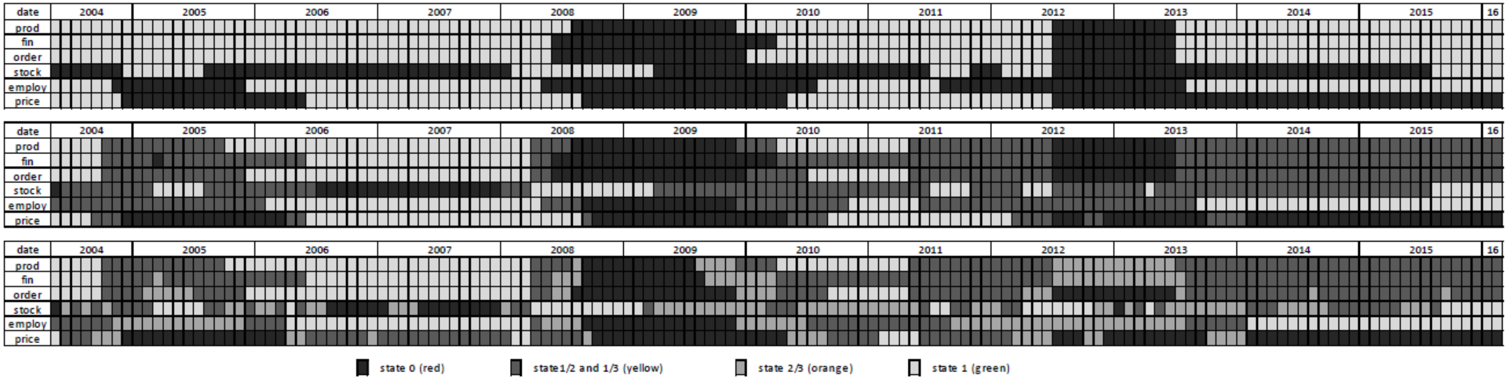

Figure 2. Results of the decomposition of univariate time series.

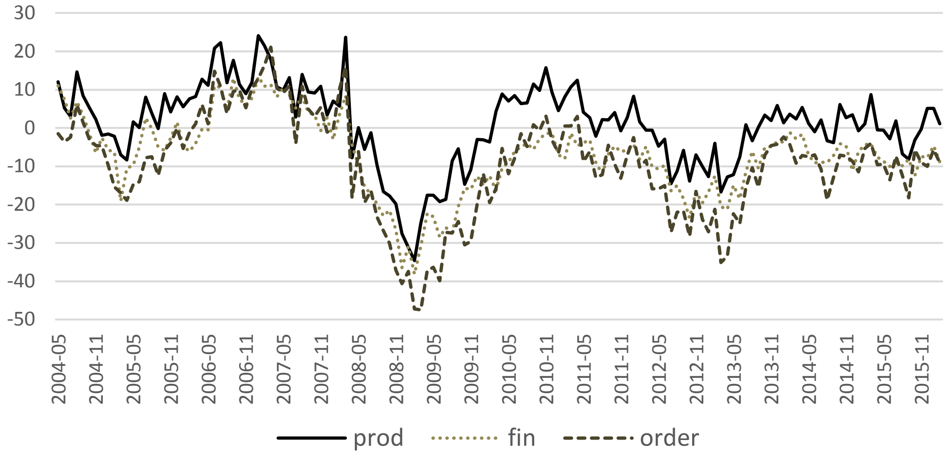

Figure 3. Original time series of the *prod*, *fin* and *order* balances.

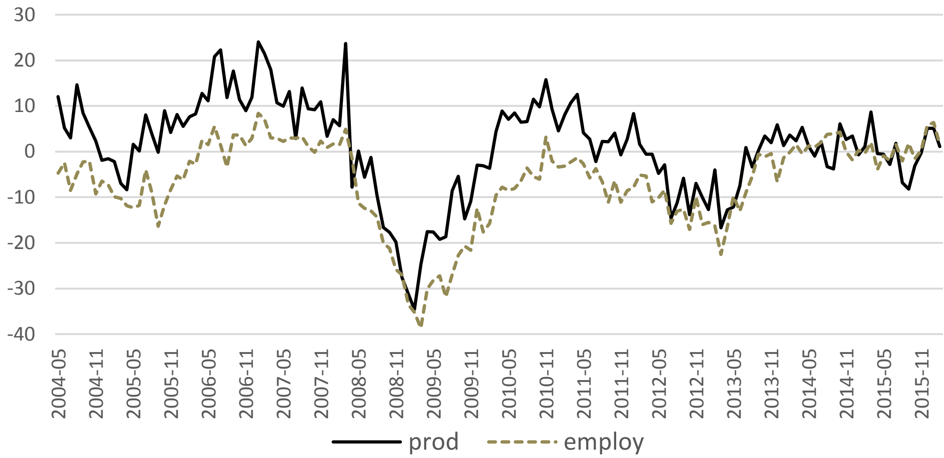

Figure 4. Original time series of the *prod* and *employ* balances.

Obviously, as shown by Figure 5, this is a reflection of the actual relationship between the balances under consideration. It does not seem possible to clearly distinguish a *leader* time series, even in the group of *prod*, *order* and *fin*.

The Viterbi paths with two states are a convenient way of pooling the balances, and they also give an opportunity to identify a leading time series, if such exists. The three-states Viterbi paths provide with even more valuable information. By comparing Figures 6 and 7 one can clearly note a difference in the pictures the two- and three-states Viterbi paths of *fin* show. However, introducing the fourth state in MS does not informationally enrich the picture a lot (see Figure 8). A similar observation can be made for the Viterbi paths of *employ* and *price* MS(1,2) (see Figures 9-11).

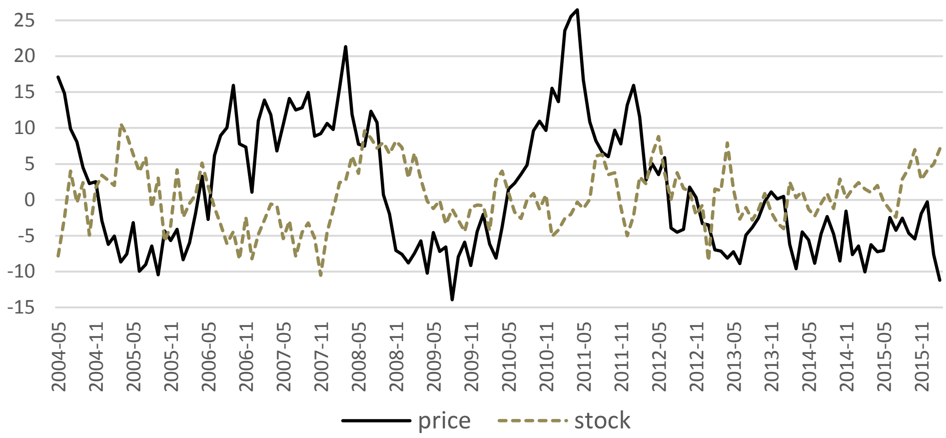

Figure 5. Original time series of the *price* and *stock* balances.

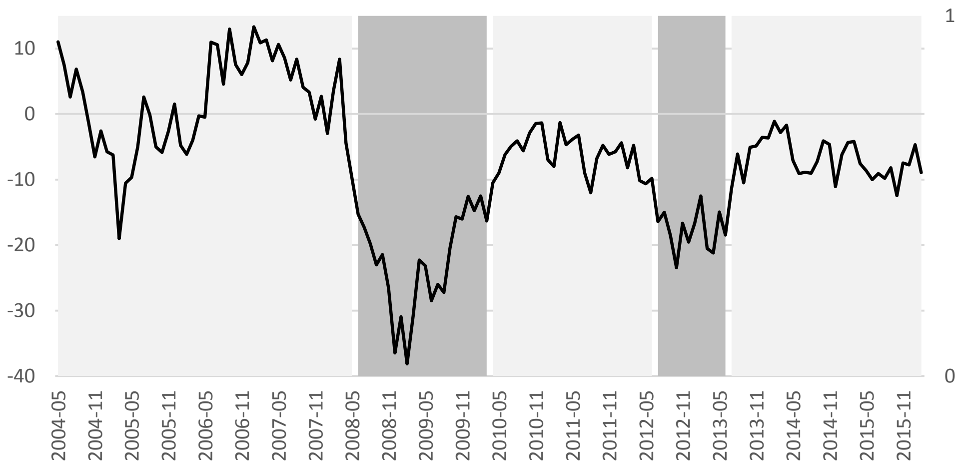

Figure 6. Original time series of *fin* vs the corresponding two-states MS.

There is no surprise that due to the higher level of decomposition, observation on the concordance of time series may change. For example, in absence of state 1 in the path of MS(1,3) for *fin* in the period April 2010 – August 2011 *prod* and *order* seem to be closer to the original time series.

The three-states Viterbi paths give an opportunity to assess the rate of change of the states associated with the high and low levels. For example, by analyzing the *prod* and *order* balances and the period February 2008 – July 2010, one can infer that the descent from state 1 to state 0, due to longer series of state ½, is slightly gentler for *prod*. On the contrary, transition from state 0 to 1 seems to be a little bit sharp (see Figure 12). Furthermore, there are periods in which the Viterbi paths seem to indicate the same range of changes (up to November 2013; see Figures 13 and 14).

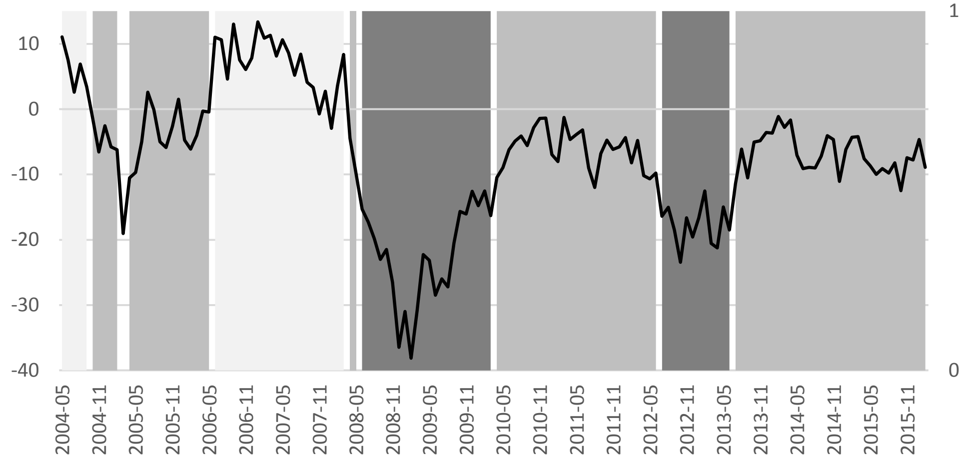

Figure 7. Original time series of *fin* vs the corresponding three-states MS.

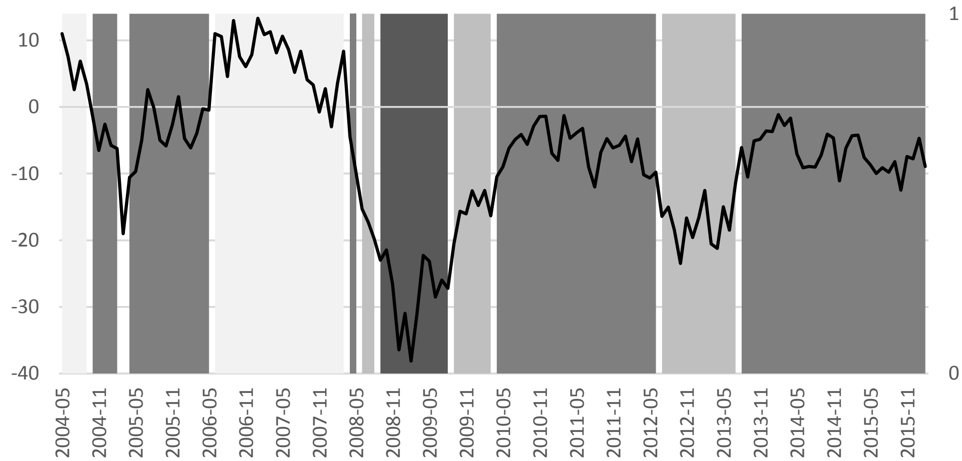

Figure 8. Original time series of *fin* vs the corresponding four-states MS.

Although the difference between the paths obtained for MS(1,4) and MS(1, 3) is not so striking as the one between the paths for MS(1,3) and

MS(1,2), the four-states paths provide crucial information as well. For example, comparing the paths for MS(1,3) and MS(1,2) for *employ* in the period up to January 2006 may be in a way a little bit misleading. A deeper insight into the path for MS(1, 4) gives a clear explanation.

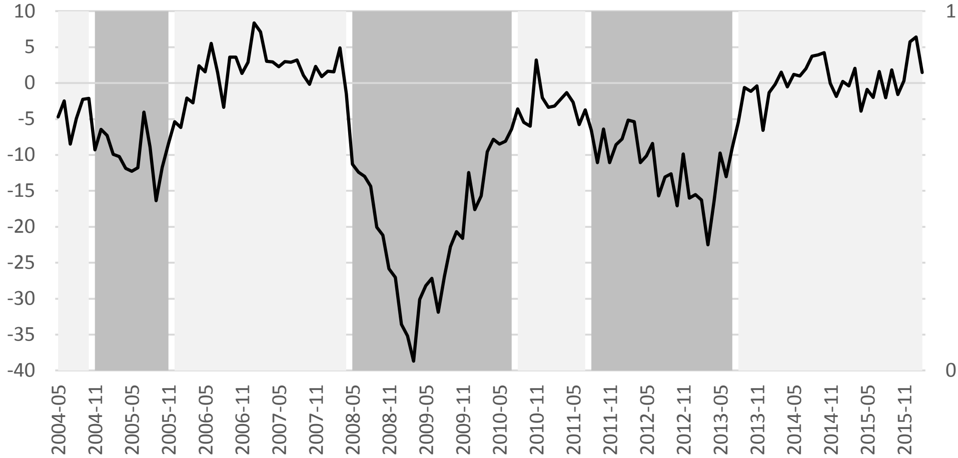

Figure 9. Original time series of *employ* vs the corresponding two-states MS.

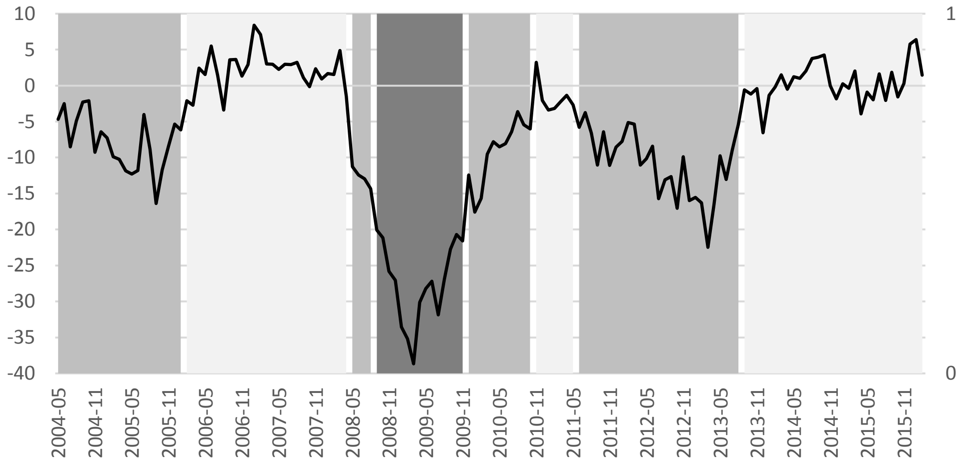

Figure 10. Original time series of *employ* vs the corresponding three-states MS.

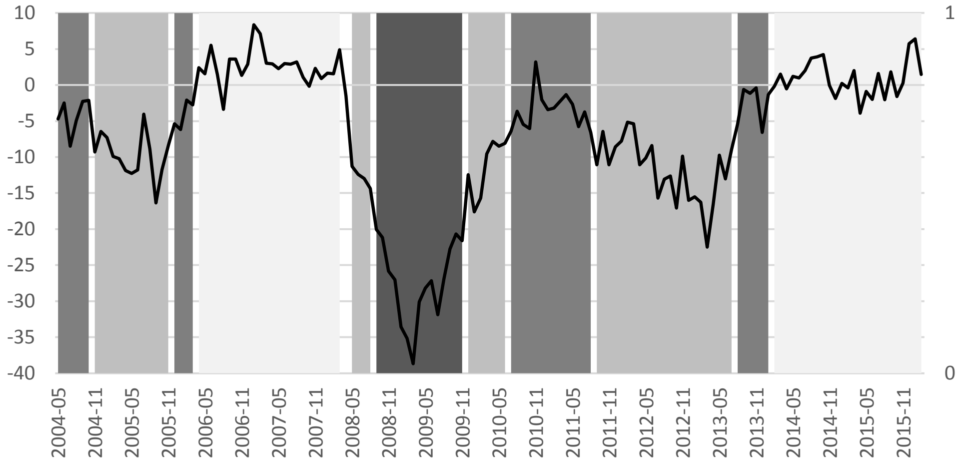

Figure 11. Original time series of *employ* vs the corresponding four-states MS.

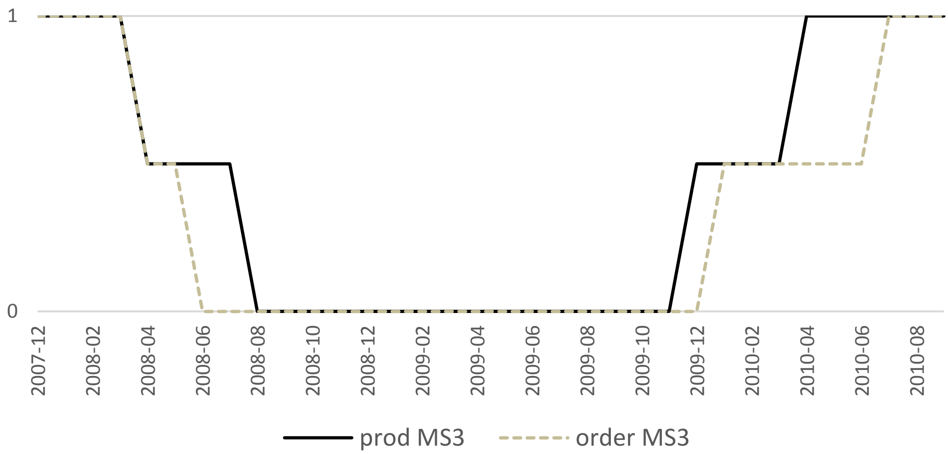

Figure 12. Part of the *prod* and *order* Viterbi paths.

3.2. Results of decomposition of bivariate time series

In this section, an example of the use of the Viterbi paths for models M(2,*k*) is given. We focus on the following pairs of the balances: (*prod*, *employ*), (*stock*, *order*) and (*order*, *price*). It turns out that not all of models M(2,*k*) fit the data. Specifically, this problem refers to M(2,2) for the pair (*order*, *price*), and M(2,4) for the pairs: (*order*, *price*) and (*stock*, *order*).

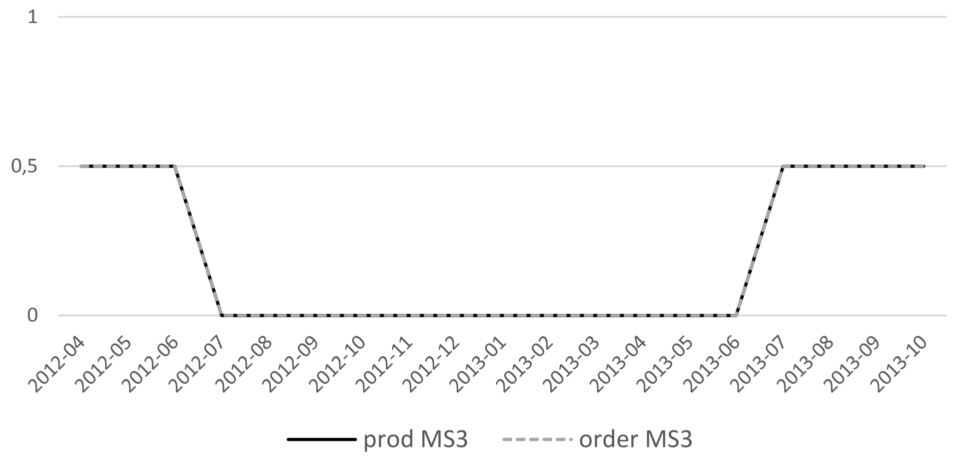

Figure 13. Part of the *prod* and *order* Viterbi paths.

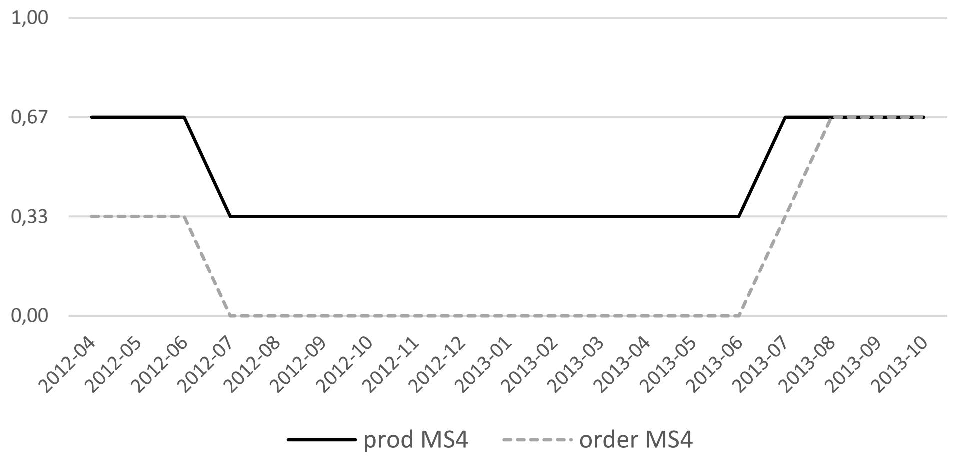

Figure 144. Part of the *prod* and *order* Viterbi paths.

In the two-dimensional case it is assumed that the hidden Markov chain reflects some common factor, which 'governs' the pairs of the observable time series. On the contrary to the one-dimensional case, interpretation of states is not obvious and should be inferred after thorough examination of Gaussian distribution mean estimates. For clarity, we decide to omit exact values and use the following symbols: ++ to denote the *high* level, + *moderate high* level, - *moderate low* level, -- *low* level, and 0 meant to be a value very close to zero. Interpretation of states of the hidden Markov chain for all models under the study is given in Table 1 (with X reserved for models that do not fit to data). The Viterbi paths obtained for the pair (*prod, employ*) are shown in Figure 15, for (*price*, *order*) in Figure 16, and for (*order*, *stock*) in Figure 17.

Table 1. Summary of the averages of normal distribution for the states of hidden Markov chain.

state	(*prod*, *employ*)	(*order*, *price*)	(*stock*, *order*)
0	(-,-)	X	(+,--)
1	(+,+)	X	(0,-)
0	(--,--)	(--,-)	(+,--)
1/2	(+,-)	(-,-)	(+,-)
1	(0,+)	(0,+)	(-,+)
0	(--,--)	X	X
1/3	(+,-)	X	X
2/3	(+,0)	X	X
1	(++,+)	X	X

Figure 15. The *prod* and *employ* Viterbi paths.

The question arises: what could Viterbi paths of bivariate time series be applied to? For example, comparing the Viterbi paths for (*prod*, *employ*) and (*order*, *price*) makes one assume that the changes which happen within a firm are perceived almost simultaneously or with a slight delay (except for the period of the Great Recession) to the corresponding changes outside that firm. Illustration is given by Figure 16. Furthermore, the Viterbi paths shown in Figure 17, suggest there is not a clear relationship between the balances related to leading economic variables and the balances connected with coincident ones.

Figure 16. The *prod* and *employ* Viterbi paths vs the *order* and *price* ones.

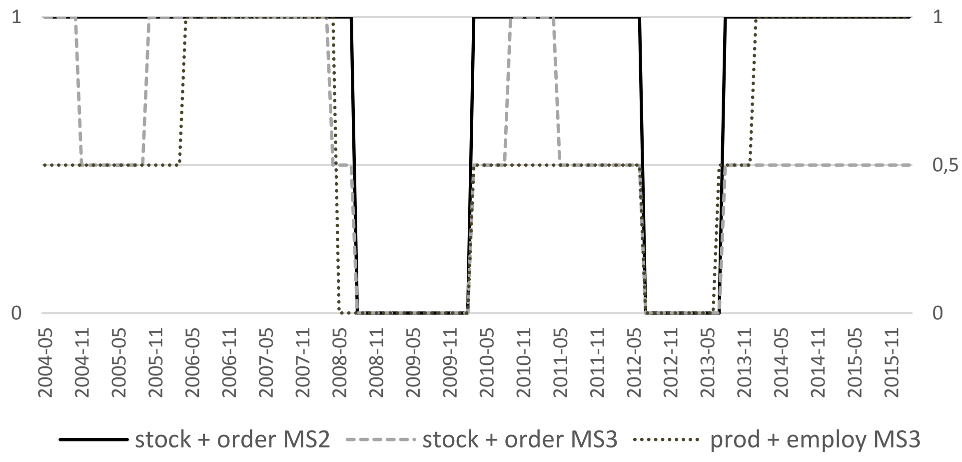

Figure 17. The *stock* and *order* Viterbi paths vs the *prod* and *employ* ones.

4. Conclusions

In this paper the application of the Viterbi paths to analyze qualitative data is examined. We focus on modeling univariate and bivariate time series. Using MS models with the conditional Gaussian distribution with two, three and four hidden states, it is possible to find decompositions of the survey balances of manufacturing production, orders, finished goods inventories, selling prices of products, employment and financial standing of manufacturers. On the whole, the Viterbi paths with two states provide a convenient way to pool time series, and to identify leading ones. We did not, however, find any leading time series in the dataset, even in the group of the balances of production, orders and financial standing, i.e., the time series with very similar Viterbi paths. The three- and four-states Viterbi paths allow assessing the change rate of the states related to *high* and *low* levels of economic phenomena under a study. Ones of the many advantages of the Viterbi paths obtained for the one-dimensional case are intuitive interpretation of results and a wide range of types of analyses that can be carried out. The Viterbi paths obtained for the two-dimensional case seem to be a promising tool too. However, they have some disadvantages. It turned out that the proposed models did not always fit to business tendency survey data. Moreover, interpretation of the hidden states might be found unclear a bit.

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