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Meanders of the Theory of Social Choice

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I would like to thank Professor Gardawski for inviting me here. I must admit I accepted this invitation rather blindly as I am a 'migrant' both in the sense of place, as I graduated from the University, and faculty, because mine is mathematics. I have been dealing with the mathematical economics for many years and I think I can say I also deal with economics. However, my approach, my views and what I consider scientific knowledge is contrary to most opinions that have been expressed here so far.

In effect, it seems to me most opinions so far have been rather old-fashioned, as regards for instance the relations of the formal model with the economic reality. The formal model is not supposed to describe the exact reality. It is there to present selected relations between various phenomena so as to facilitate research on them. For example the representative agent is an instrument we use for analysis but it is not something that describes reality or an average. The theory of general equilibrium is not just a theory of historical significance. It is used to explain not only microeconomic problems but also macroeconomic and financial (see: Borglin 2004 or LeRoy, Werner 2001). A model can have many equilibriums, also stable ones. The concept of bifurcation allows to model the transition from one state of equilibrium to another changing parameters. Depending on what the model is, they can describe for instance a technological process or employment. And it is not like we always stay in one state of equilibrium. The models with bifurcation notion have been used since the 1990s. There is a catastrophe theory which also explains certain changes. The literature on this subject is for example A. Jakimowicz (2003), Gandolfo (2005), Medio

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(1999), Day (1999). All the abovementioned models require the knowledge of fairly advanced mathematical methods, but this is exactly how the theory of economics evolves.

I fervently object to what was said by doctor Czarny on the methodology of economics. My objection is simply that I do not consider his approach as a scientific one. I should say it is rather a kind of scientific populism. In my opinion Doctor Czarny presented a range of quite randomly selected quotes which were to illustrate some impressive theses that sadly had nothing to do with the serious methodology of economics. The methodology of economics is a very complex matter and difficult too, because economics deals with mathematics, psychology, sociology, and comes up with scientific results that happen a few times in a century. This simply cannot be nice and easy. But it obviously is learnable.

Professor Gardawski asked me here to talk about the theory of social choice, which is one of my scientific areas. The theory of social choice (in Polish we also call it the group choice) tells us how we aggregate individual preferences or decisions into one group decision. It dates back as a separate area of scientific research to the beginning of 1950s and the publication of Social Choice and Individual Values by Kenneth J. Arrow (second edition, Wiley, 1963). Since then the theory of social choice has been actively developing. It has recently gained more significance because in times of modern technology various kinds of voting (that is group decisions) have become a way of social communication. We vote not only in general elections. We vote when we choose contestants in a TV show or when we pick the name for a street. There is an international scientific society, called the Society for Social Choice and Welfare, of which I am a member. They have their journal 'Social Choice and Welfare' issued in cooperation with Springer Verlag. And every two years organise a congress in cooperation with universities from across the world. The last one was a year ago. During the congress the speakers presented 300 papers on such problems as: coalition formation, market mechanisms, strategyproofness, allocation and fairness axioms, house allocation, distributive justice and equality, group decisions, auctions, cost sharing and assigment, claims problem, land division, voting rules, political campaigns, proportional representation and apportionment, jury problems, power indices, political competition, solutions for cooperative games, public goods, inequality, measurement of welfare and poverty.

Speaking of the theory of social choice I'd like to refer to the earlier mentioned relation between the formal model and the reality. Nearly twenty centuries ago in Ancient Rome, Pliny the Younger presided over an assembly that was to adjudicate in the case against the freedmen of a certain patrician. The patrician was found dead

and nobody knew who killed him. The freedmen were the suspects but there was no proof of their guilt. The assembly had to decided what was to become of the freedmen. Once they have been tortured the choice was either to let them go free or sentence them to exile or death. Each of these alternatives had the same number of advocates, the mayhem ensued and Pliny the Younger who presided over the assembly in all this mess forgot as all the others how decisions such as this one should be made.

So they had a situation in which they didn't know what to decide. Are those who want to condemn to exile closer to the advocates of the death penalty or those who want the freedmen let go free? If they were more in favour of the death penalty, then that group would prevail and the freedmen would be executed. On the other hand, if the exile option group were to the group who were in favour of freeing of the accused then that fraction would prevail. It was very hard for them to make any decision. In the end Pliny proposed to vote by simple majority. He himself was in favour of letting the accused go free, and since the count showed the advocates of the absolution were slightly more numerous than the two other groups finally the freedmen were allowed to go free. The story I told you is just a summary. Pliny's letter in which he describes this event is a few pages long and it is very hard to figure it out. The letter with the comments can be found in the "Decyzje" (2007: 117-130).

Why is it easier for me to figure this out? Because in the 18th century Marquis de Condorcet introduced the notion of Condorcet's paradox, which was a model of a situation, named after him. In this situation the preferences of the voters are in conflict with each other. The theory we know says there is no good solution to this situation. You can manipulate the vote by the choice of order of the comparison in pairs. Different orders of comparison lead to different results. Unless Condorcet's paradox has clear cut rules on how the assembly should be presided gives the president certain possibility of manipulating the order of voting. Pliny who was a smart man, seeing the ambiguity, manipulated the method of vote so that the option he was in favour of won.

This was the first description of a vote manipulating in history. Pliny's letter is the first reflection on the methods of voting. Today I would formulate the problem in a different manner. I would say the voters have preferences and that they can put in order their preferences of the verdict proposals. In my saying that the voters have preferences I am applying an idealisation. If I asked each of those Roman senators what were his preferences he would not know what I mean. I would have to explain to him that it does not matter what he thinks was the cause of death of the patrician, what matters is that there are three proposals of the verdict and he should arrange them in order, keeping certain rules of rationality. And every single one of those

senators would have to do this. I would have to explain it to them that with certain arrangements of individual preferences different methods of voting give different results. Then I would surely hear that I should not mess with their heads and come up with a 'decent' universal method of voting. Only today we already know that there is no such method. This result, also known as Arrow's impossibility theorem or Arrow's dictator theorem, requires formal context and use of methods of the 20th century logic and theory of sets zmiana. The mathematical model of the social choice enables to pick out the essence of every specific problem and analyse the conditions of its occurrence and the number of solutions. There is no need to run thousands of votes and studying them under different aspects. We have a certain pattern of description and analysis. Of course, the instruments that are used in this patter are an idealisation, like for instance the preferences. As much as Pliny and the Roman senators knew how to arrange the three possible verdicts on the involvement of the freedmen in the death of the patrician, although psychologically this would not be the easiest, the arrangement of such thing as brands of beer from the best to the least liked could be a mind blower to an average consumer. Not everybody knows if they like Tyskie better than Lech. Nevertheless, in describing consumer behaviours on the market we assume they are in fact able to arrange beer brands in order of personal preference. In this model we deal with an ideal consumer. Accepting the relation of individual preferences with defined assumptions makes it possible to use the function of utility and look for a solution to the optimization problem.

The famous Arrow's impossibility theorem tells us one would like to have a method of group choice, for instance some sort of voting, that would meet certain, apparently obvious, conditions. Only, when you formally write down all these conditions and compare them, it turns out there is no perfect method. And this is what the formal approach is giving us. The formal approach can tell you that some sets of norms (the conditions a method should fulfil are certain norms imposed on the method) can be inconsistent. The formal approach can tell us there is no method that would satisfy this set of norms in a given situation. It points to a certain relativism of norms and at the same time introduces order to the theory. We know what to expect of the theory.

As I said before, there are many areas that deal with the theory of social choice. Apart from this, we do more and more experiments. We want to know to what extent our normative theories find experimental confirmation. If they don't we ask ourselves which axioms may have weaker forms. We also deal with the bounded rationality of our agents, who make decisions. Based on these experiments, we build various other theorems which even refer in some way to social psychology. Therefore, the theory of

social choice is not purely normative. It builds its subsequent theorems in relation to the results of experimental research, trying to figure out how the society is working, although we do realise it is an infinite asymptote.

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