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Commodity and Financial Market Linkages: Granger Causality Insights from Rare Earths, Crude Oil, and Equities

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Abstract

The article begins with a comprehensive review of the literature, emphasising empirical evidence on the interdependencies between rare earth element returns and other financial and commodity variables. Subsequently, two econometric models are specified and estimated to identify causal relationships among the returns of selected rare earth elements, crude oil, and an index representing exposure to the broad equity market. The empirical analysis incorporates the interpretation of impulse response functions (IRFs) and forecast-error variance decomposition (FEVD), which are integral components of VAR-based modelling. Finally, the findings are synthesised and conclusions are presented.

Keywords: vector autoregression, lead, zinc, EURO STOXX 50, WTI crude oil, Granger causality, impulse response function, forward-error variance decomposition

JEL Classification: C22; G15; G17.

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Introduction

Determining the relationships among economic variables remains one of the fundamental challenges in economics. Researchers often aim to understand how changes in certain factors influence key indicators such as GDP, inflation, employment, and industrial output, or explore the mechanisms through which these interactions shape overall economic performance and long-term growth.

In financial research, the relationships among variables are more complex and multidimensional. This complexity arises from the cyclical nature of financial variables, the disproportionality of effects relative to causes, and the feedback mechanisms that frequently characterise such interactions. Moreover, financial data are often influenced by factors that are difficult to quantify, reflecting, among other things, the behavioural nature of financial markets. These challenges become evident when analysing linkages between the prices of rare earth elements and the quotations of financial or commodity assets, derivatives, cryptocurrencies, exchange rates, or macroeconomic indicators.

Regression analysis is commonly employed to explore such relationships. However, this approach suffers from a key limitation – it cannot establish causality between the variables included in the model. Regression models assume, ex ante, which variable is explanatory and which is dependent, and then estimate parameters to forecast future values of the dependent variable. In this sense, regression may be viewed as an extension of Pearson's correlation. An alternative approach is causality analysis, typically implemented through atheoretical vector autoregressive (VAR) models, which impose more stringent requirements on the statistical properties of time series.

The primary objective of this study is to examine Granger causality among the returns of selected rare earth elements, i.e. lead and zinc, WTI crude oil, and the EURO STOXX 50 index. Identifying causal linkages between these assets is crucial, as it may facilitate the formation of heterogeneous asset classes and, consequently, support the construction of diversified portfolios of financial instruments. Furthermore, the findings may contribute to the development or refinement of investment strategies that incorporate the instruments under analysis.

The article begins with a comprehensive review of the literature, focusing on empirical evidence concerning the interdependencies between rare earth element returns and other financial and commodity variables. Subsequently, two econometric models are specified and estimated to identify causal relationships among the returns of selected rare earth elements, crude oil quotations, and an index reflecting exposure to the broad equity market. The empirical analysis includes the interpretation of impulse response functions (IRFs) and forecast-error variance decomposition (FEVD), which are integral components of VAR-based modelling. Finally, the results are synthesised and conclusions are drawn.

Literature review

Studies examining price interdependencies between metals and other variables can be broadly classified into four groups.

The first group of studies focuses on relationships among individual metals. This research stream includes, for example, Ciner (2001), who demonstrated that gold prices Granger-cause silver quotations. These findings were extended by Krawiec and Górska (2015), who confirmed causal linkages for a broader set of metals, including palladium and platinum. Based on their results, they concluded that silver and platinum quotations, as well as palladium and silver, exhibit significant interdependencies. A different set of metals and methodology was employed by Śmiech & Papież (2012), whose analysis revealed that between 2000 and 2003, copper prices Granger-caused gold, silver, and platinum quotations, whereas after 2004, gold, silver, and copper prices Granger-caused platinum quotations. Another perspective was offered by Baselou et al. (2014), who examined all non-ferrous metals listed on the London Metal Exchange between 2000 and 2013 using a two-variable VAR model. Their findings indicate that aluminium prices influence copper quotations, lead prices affect nickel quotations, and copper prices determine lead and nickel quotations. While such analyses enhance understanding of metal market dynamics, they remain limited by the absence of a broader perspective on interdependencies within national economies. For instance, gold prices may serve as useful predictors of inflation, while industrial metals such as steel and copper can act as indicators of economic growth. Subsequent research was undertaken by Apergis and Apergis (2017), who established the existence of a long-run relationship between rare earth prices and renewable energy consumption. Gao & Liu (2024) adopted a considerably broader set of variables and demonstrated that the rare earth metals market functions as a net spillover recipient from the base metal, clean energy, and ESG markets, which should be regarded as the three principal net risk emitters. Furthermore, their findings indicate that financial conditions and investor sentiment exert a significant influence on the strength of these interconnections. Additionally, the frequency of data employed has a material impact on the robustness and nature of the results obtained. Zheng et al. (2022) conducted a different type of study, examining volatility spillovers between crude oil, renewable energy, and high-technology markets. Their results showed that there are volatility spillovers between renewable energy and high-technology stock markets, and that the renewable energy market is more closely linked to high-technology than to crude oil.

The second group of studies examines relationships among metals that are jointly consumed due to their shared application in specific technologies. Pradhananga (2016) notes that prices of metals with interdependent uses may exhibit mutual transmission effects. Similar conclusions were drawn by Rossen (2015), who analysed long-term price series for steel, iron, and molybdenum. Shammugam et al. (2019) confirmed this hypothesis, observing that joint consumption of primary metals influences their price linkages. They also emphasised that metals jointly consumed exhibit stronger interdependencies than those jointly produced. Tok (2025) extended this line of research by exploring the relationship between the oil market and metals critical for renewable energy technologies. The findings indicate that copper and aluminium are the most significant information transmitters, while oil prices, particularly Brent and WTI, become more sensitive to metal markets during periods of crisis. Moreover, as the energy transition accelerates, critical metals increasingly affect commodity markets, influencing energy pricing dynamics among other factors.

The third group of studies concerns relationships among metals jointly produced. The tendency of price paths to move together in such cases may result from similar reactions to specific market information. According to Butti and Sapir (1998), a good example involves primary metals and those extracted as by-products. Here, the price of one metal depends on the market conditions of another, which may be influenced by numerous factors. One of the earliest authors to address this issue was Campbell (1985), who, using a sign test, demonstrated that gold and silver quotations, as well as lead and zinc, are interrelated. Similar studies, though employing different methodologies, were conducted by Kim and Heo (2012). Analysing metals jointly mined and subsequently used in solar panel production, they showed that the prices of metals accompanying zinc and copper extraction, namely germanium, indium, cadmium, and selenium, are interconnected. These relationships are unidirectional, running from zinc to cadmium and germanium, and from copper to selenium. Correlations between primary metals and by-products were also examined by Afflerbach et al. (2014), whose findings differed from those of previous studies. Of the thirteen metal pairs considered, only germanium and zinc, and tin and copper exhibited significant linkages, whereas pairs such as aluminium and gallium or cobalt and nickel displayed weak interdependencies. Su et al. (2025) analysed similar issues, focusing on the dynamic relationship between copper and its by-product metals, cobalt and nickel, within the upstream raw material market for new energy vehicles. Their findings indicated that spillover effects among the copper, cobalt, and nickel markets vary over time. Moreover, the copper market is the predominant source of volatility for the other two markets, whereas the cobalt market contributes minimally. Within these relationships, Su et al. (2025) also found that both copper and nickel act as net contributors of shocks to the other markets, while cobalt primarily receives shocks.

The fourth group of studies focuses on the relationships between rare earth element prices and the quotations of other financial instruments or various micro- and macroeconomic variables. This research stream is represented, among others, by Reboredo and Ugolini (2020), who analysed the transmission mechanism between rare earth stocks and base metals, gold, clean energy, oil, and global MSCI stock markets. Their findings indicate that price connectedness between rare earth and other stock markets varies across volatility regimes. In a low-volatility regime, rare earth stocks are closely linked to the base metals market, both receiving and transmitting substantial price spillovers, while their connections with clean energy, gold, oil, and general stock markets remain weak. In contrast, in a high-volatility regime, rare earth prices exhibit stronger co-movement with price fluctuations in clean energy, oil, and broader stock markets. Song et al. (2021) also examined the linkages between rare earth markets and other segments of the financial and commodity markets. Their findings showed that volatility connectedness between the rare earth stock market and clean energy, global equity, base metals, gold, and crude oil markets is generally stronger than return connectedness. During the COVID-19 outbreak, the rare earth index displayed close interdependence with clean energy, global equity, and oil indices, while remaining primarily a receiver of return and volatility throughout the entire period. Notably, during the outbreak of the pandemic, the rare earth stock index showed strong linkages with clean energy and global equity. Furthermore, the volatility of the rare earth stock index exhibited a pronounced interdependence with crude oil price volatility.

Data and methodology

Data

The empirical analysis was based on data encompassing four categories of variables. The first two comprised lead and zinc prices, which serve as benchmark indicators representative of the global market for these non-ferrous metals. These prices are determined by the largest exporter of each commodity. The time series were complemented by historical values of the EURO STOXX 50 index, which tracks leading companies across Eurozone supersectors, thereby ensuring a diversified and liquid portfolio. The index is weighted according to free-float market capitalisation, with a maximum allocation of 10 per cent per constituent. It functions as the underlying instrument for over 25 billion euros in ETF assets, while futures and options on the index rank among the most actively traded equity derivatives on Eurex. Moreover, more than 160,000 structured products are linked to the EURO STOXX 50.

In addition to historical lead and zinc prices and the EURO STOXX 50 index, the study incorporated global WTI crude oil benchmark prices, which are determined by the largest global exporter of the commodity. The dataset comprises monthly observations spanning the period from June 2011 to June 2025. All data were sourced from the FRED database maintained by the Federal Reserve Bank of St. Louis, except for historical index quotations, which were obtained from the Standard & Poor's website.

Descriptive statistics for the individual variables are presented in Table 1.

Table 1. Descriptive statistics of the time series of log returns of the analysed variables

Variable	Mean	Min.	Max.	Std. dev.	Pr (skewness)	Pr (kurtosis)	Prob >chi2
d.ln.lead.	-0.00120	-0.1544	0.1376	0.0459	0.1182	0.2840	0.1621
d.ln.zinc	0.00122	-0.1624	0.1451	0.0567	0.1222	0.5007	0.2367
d.ln.EUROSTOXX50	0.00341	-0.2197	0.1548	0.0500	0.0007	0.0006	0.0001
d.ln.WTI	-0.00231	-0.5465	0.5280	0.1083	0.0001	0.0000	0.0000

Source: authors' own elaboration

From the data presented in Table 1, it can be concluded that the variable d.ln.WTI exhibits the lowest mean value while simultaneously revealing the highest standard deviation. In contrast, the variable corresponding to the logarithmic rates of return on the EURO STOXX 50 index records the highest mean, whereas d.ln.lead demonstrates the lowest standard deviation. Furthermore, the p-values of 0.1182 and 0.2840 for d.ln.lead and 0.1222 and 0.5007 for d.ln.zinc suggest that the skewness and kurtosis of these variables do not significantly deviate from those of a normal distribution at the 5% significance level. Consequently, based on skewness and kurtosis, the null hypothesis of a normal distribution cannot be rejected for these variables. Conversely, for the remaining variables, i.e. d.ln.EUROSTOXX50 and d.ln.WTI, the analysis of skewness and kurtosis provides clear evidence that the null hypothesis of a normal distribution should be rejected.

Methodology

This study adopts an approach grounded in the vector autoregressive model with lags, i.e. model, originally formulated by Sims (1980) and subsequently extended by Granger (1980). The model is applied to investigate the interdependencies between logarithmic returns of selected non-ferrous metals (lead and zinc) and relative changes in the logarithmic prices of WTI crude oil and the logarithmic values of the EURO STOXX 50 index.

In its standard form, the model assumes that all variables are treated *a priori* as jointly endogenous. This means that each variable is influenced not only by the lagged values of all variables included in the model but also by its own stochastic component and stochastic components of all other variables.

For the purposes of this analysis, the following model is proposed:

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1\mathbf{x}_{t-1} + \mathbf{b}_2\mathbf{x}_{t-2} + \cdots + \mathbf{b}_p\mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t$$

where: $\mathbf{x}_t = (x_{1,t}, \dots, x_{n,t})'$ is a vector of endogenous variables from Table 1, $\mathbf{b_0}$ is an $(n+1)$ vector of parameters, $\mathbf{b}_1$ through $\mathbf{b}_p$ are $(n \times n)$ matrices of coefficients corresponding to vectors of $\mathbf{x}_{t-1}$ through $\mathbf{x}_{t-p}$, $\mathbf{x}_{t-p}$ is an $(n \times 1)$ vector of regressors with p lags, and $\boldsymbol{\varepsilon}_t$ denotes an $(n \times 1)$ unobservable zero mean white noise vector of disturbances (serially uncorrelated and independent) with a time-invariant covariance matrix $\sum$. It means that: $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$, $E(\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t') = \sum$, $E(\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_p') = \mathbf{0}\ \forall t \neq p$. Furthermore, it is assumed that $\boldsymbol{\varepsilon}_t \sim iid$.

As part of the study, the data were organised so that the time series related to the quotations of a single non-ferrous metal was always included in the dataset. Consequently, two models were specified: the first incorporated logarithmic return of lead, WTI crude oil benchmark, and EURO STOXX 50 index (model 1); the second included logarithmic returns of zinc, WTI crude oil benchmark, and EURO STOXX 50 index (model 2). From a technical perspective, this implies that in each model, the number of rows in the vectors and matrices in the equation associated with model was set at three, i.e. .

Several factors highlight the limitations of the chosen approach. First, applying the model assumes that all time series are stationary. Second, the series must not exhibit cointegration; if cointegration is present, a VEC model should be used instead. Third, it is important to note that the model is atheoretical, meaning it is not based on any underlying economic theory. Finally, the standard model ignores potential issues such as non-linearity, conditional heteroskedasticity, and structural breaks in parameters.

To examine the interdependencies among the selected variables, a structured six-step procedure was applied:

1. Determining stationarity of the time series.
2. Selecting the optimal lag length.
3. Testing for cointegration among the variables.
4. Conducting Granger causality tests to determine the direction of the relationships.
5. Investigating stability and autocorrelation of residuals.
6. Plotting impulse response functions (IRFs) and performing forecast-error variance decomposition (FEVD).

Results

Following the methodology outlined earlier, the first step involved examining the stationarity of the time series. Stationarity is a critical aspect of the proposed approach, as the model requires all variables to be integrated of order one, i.e., . To verify this, the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests were applied. Although these tests are widely used in time series analysis, they may produce unreliable results when applied to small samples. Given that the dataset comprises 169 observations, the assessment of stationarity was based on these tests. The results are presented in Table 2.

The data reported in Table 2 indicate that all variables included in the model are non-stationary in levels but become stationary after first differencing (at the 1% significance level). This implies that each time series is integrated of order one, . It is worth noting that this property holds regardless of which unit root test is employed.

Table 2. Unit root tests of the variables.

Variable	Augmented Dickey–Fuller (ADF) Test		Phillips–Perron (PP) Test		Stationarity Order
	At level	At 1st difference	At level	At 1st difference	
ln.lead.	-3.229	-12.141***	-3.358	-12.117***	
ln.zinc	-1.778	-11.222***	-2.033	-11.234***	
	-0.895	-13.078***	-0.778	-13.150***	
ln.WTI	-2.309	-9.875***	-2.393	-9.549***	

*** indicates significance at the 1% level

Source: authors' own elaboration

The data reported in Table 2 indicate that all variables included in the model are non-stationary in levels but become stationary after first differencing (at the 1% significance level). This implies that each time series is integrated of order one, . It is worth noting that this property holds regardless of which unit root test is employed.

In the next stage of the analysis, the optimal lag order was determined. For this purpose, two criteria were applied: the Final Prediction Error (FPE) and the Akaike Information Criterion (AIC). The results are presented in Table 3.

Table 3. Optimal lag length selection.

Models	Lag	FPE	AIC
Model 1	0	4.8e-08	-8.33991
	1	4.6e-08	-8.38367
	2	4.6e-08	-8.38732
	3	4.5e-08	-8.41008
	4	4.3e-08	-8.44670
	5	4.3e-08*	-8.45345*

Model 2	0	7.8e-08	-7.84848
	1	7.3e-08	-7.91668
	2	7.3e-08	-7.92613
	3	7.5e-08	-7.89697
	4	7.0e-08*	-7.95879*

* indicates the optimal lag length based on the criterion.

Source: authors' own elaboration

The key conclusion that can be drawn from the data presented in Table 3 is that, in model 1, the optimal lag order is five, whereas in model 2 it is four. This finding provides an important basis for later phases of the analysis, including model estimation and the investigation of Granger causality.

In the subsequent stage, the procedure focused on identifying long-run relationships among the analysed variables. For this purpose, the Johansen cointegration test was conducted. As cointegration analysis applies to non-stationary variables, the input data were transformed into logarithmic form of the corresponding values rather than their first differences. The results are reported in Table 4.

Table 4. Johansen test for cointegration.

	Model 1	**Model 2**	
Maximum Rank	**Trace Statistics**	**Trace Statistics**	**Critical Value (5%)**
0	21.0489*	14.2809*	29.68
1	6.5618	6.2484	15.41
2	0.2759	1.5085	3.76
Maximum Rank	**Maximum Statistics**	**Maximum Statistics**	**Critical Value (5%)**
0	14.4871	8.0325	20.97
1	6.2859	4.7399	14.07
2	0.2759	1.5085	3.76

* indicates order of cointegration.

Source: authors' own elaboration

The data presented in Table 4 indicate that both the trace statistic and the maximum statistic do not exceed the corresponding critical values for zero cointegrating equations (a maximum rank of zero). On this basis, it can be inferred that there is no cointegration among the variables considered in either model 1 or model 2. This implies the absence of long-term relationships between ln.lead, ln.EUROSTOXX50 and ln.WTI, as well as between ln.zinc, ln.EUROSTOXX50 and ln.WTI.

In the next stage of the analysis, the Granger causality test was conducted. The results obtained for models 1 and 2 are reported in Table 5.

Table 5. The Granger causality tests for models 1 and 2.

Models	Equation	Excluded	chi sq.	df	p-value
Model 1	d.ln.lead	d.ln.EUROSTOXX50	15.970	5	0.007
	d.ln.lead	d.ln.WTI	21.409	5	0.001
	d.ln.lead	ALL	32.759	10	0.000
	d.ln.EUROSTOXX50	d.ln.lead	13.477	5	0.019
	d.ln.EUROSTOXX50	d.ln.WTI	20.009	5	0.001
	d.ln.EUROSTOXX50	ALL	37.049	10	0.000
	d.ln.WTI	d.ln.lead	17.843	5	0.003
	d.ln.WTI	d.ln.EUROSTOXX50	20.014	5	0.001
	d.ln.WTI	ALL	28.664	10	0.001
Model 2	d.ln.zinc	d.ln.EUROSTOXX50	9.9017	4	0.042
	d.ln.zinc	d.ln.WTI	13.329	4	0.010
	d.ln.zinc	ALL	23.012	8	0.003
	d.ln.EUROSTOXX50	d.ln.zinc	8.3350	4	0.080
	d.ln.EUROSTOXX50	d.ln.WTI	19.981	4	0.001
	d.ln.EUROSTOXX50	ALL	28.141	8	0.000
	d.ln.WTI	d.ln.zinc	14.192	4	0.007
	d.ln.WTI	d.ln.EUROSTOXX50	11.385	4	0.023
	d.ln.WTI	ALL	23.661	8	0.003

Source: authors' own elaboration

Based on the data presented in Table 5, four key conclusions can be drawn for model 1. First, d.ln.lead is Granger-caused by the lagged values of d.ln.EUROSTOXX50 and d.ln.WTI. Second, d.ln.EUROSTOXX50 is Granger-caused by the lagged values of d.ln.lead and d.ln.WTI. Third, d.ln.WTI is Granger-caused by the lagged values of d.ln.lead. Fourth, all these causal relationships are bidirectional, indicating that the dependencies among d.ln.lead, d.ln.EUROSTOXX50, and d.ln.WTI exhibit bidirectional linkages. Consequently, it can be concluded that any variable from Model 1 is informative for forecasting the values of the remaining variables within the model. Alternatively, this suggests that including any of these variables enhances the predictive accuracy and overall robustness of forecasts generated by model 1.

With regard to model 2, the results indicate that d.ln.EUROSTOXX50 and d.ln.WTI are Granger-causes of d.ln.zinc. Furthermore, d.ln.EUROSTOXX50 is Granger-caused by d.ln.WTI, while d.ln.WTI is Granger-caused by the lagged values of both d.ln.zinc and d.ln.EUROSTOXX50. It is worth emphasising that all causal relationships are bidirectional except for the causality between d.ln.zinc and d.ln.EUROSTOXX50, which is unidirectional.

Subsequently, as part of the post-estimation procedure, tests for residual autocorrelation and model stability were conducted. The results are presented in Tables 6 and Figure 1.

Table 6. LM test for the presence of the ARCH effect in residuals.

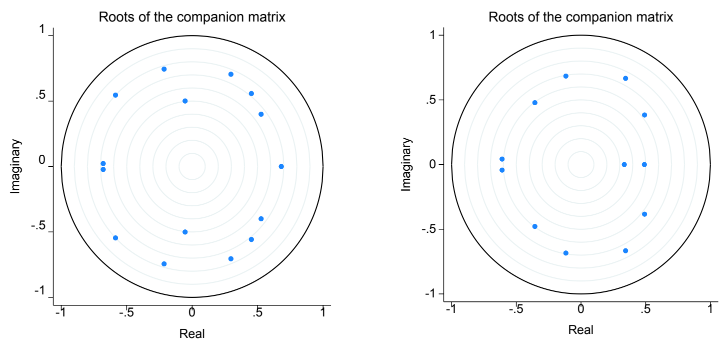
	Model 1		Model 2	
Lag	**chi sq.**	**p-value**	**chi sq.**	**p-value**
1	5.5923	0.77992	19.3614	0.02229
2	10.3309	0.32436	7.4063	0.59489
3	3.6070	0.93533	9.6696	0.37789
4	14.3831	0.10934	10.3972	0.31930
5	12.6535	0.17892	14.0548	0.12039

Source: authors' own elaboration

Based on the information in Table 6, it can be concluded that there is no evidence of autocorrelation in the residuals for any of the five lag orders tested in either model 1 or model 2. This suggests that the models are not misspecified.

Figure 1. Eigenvalues of the companion matrix for: a) model 1 and b) model 2.

a) b)



Source: authors' own elaboration

At the same time, as shown in Figure 1, for both models all unit roots fall within the unit circle, indicating that the models are both reliable and stable.

In the next stage of the research, the IRFs are established. It is important to emphasise that each function illustrates the effect of a shock in an endogenous variable on the whole system of equations in the model. In all figures presented in Appendices A and B. The shaded area represents the interval of one standard deviation around the solid line, which shows the response of the endogenous variable to the impulse.

The empirical results indicate that, in both models, shocks to individual variables exhibit a tendency to decay over time. Specifically, in model 1:

- The effect of a shock to d.ln.WTI on subsequent values of d.ln.EUROSTOXX50 is greater than the reverse effect. At the same time, d.ln.EUROSTOXX50 exhibits a negative response from the first to the third lag following a shock to d.ln.WTI, then shifts to a positive response in the fourth lag. From the fifth lag onwards, the effect converges towards zero.
- The effects of a shock to d.ln.EUROSTOXX50 on future values of d.ln.lead, as well as the effects of a shock to d.ln.lead on future values of d.ln.EUROSTOXX50, are both moderate.
- d.ln.lead demonstrates a positive response in the first, second, and third lags, and a negative response in the seventh lag, following a shock to d.ln.WTI. Conversely, d.ln.WTI exhibits a positive response in the first and fifth lags and a negative response in the fourth lag following a shock to d.ln.lead.

In model 2:

- As previously, the effect of one-standard-deviation shock to d.ln.WTI on future values of d.ln.EUROSTOXX50 is greater than the reverse effect. At the same time, d.ln.EUROSTOXX50 exhibits a negative response from the first to the third lag following a shock to d.ln.WTI, then shifts to a positive response in the fourth lag. From the fifth lag onwards, the effect quickly vanishes.
- The effect of a shock to d.ln.EUROSTOXX50 on future values of d.ln.zinc, as well as the effect of a shock to d.ln.zinc on future values of d.ln.EUROSTOXX50, are both moderate.
- d.ln.zinc demonstrates a positive response in the first and second lags following a shock to d.ln.WTI, while d.ln.WTI exhibits a positive response in the first and third lags and a negative response in the fourth lag following a shock to d.ln.zinc.

Finally, FEVD is performed. It should be noted that, while IRFs trace the effect of a shock to an endogenous variable across the entire system of equations in the model, variance decomposition is employed to separate the variation in an endogenous variable into the component shocks to the model. The results of the variance decomposition are presented in the Appendices C and D.

An examination of Appendices C and D indicates that, in both models, the predominant share of variation in d.ln.lead, d.ln.zinc, d.ln.EUROSTOXX50, and d.ln.WTI is attributable to shocks originating from these variables. Although the contribution of other variables to the variation in d.ln.lead, d.ln.zinc, d.ln.EUROSTOXX50, and d.ln.WTI tends to increase over time, it remains relatively minor throughout the entire period.

Conclusions

The primary objective of this study was to examine Granger causality among the returns of selected rare earth elements (lead and zinc), WTI crude oil, and the EURO STOXX 50 index. The analysis employed two vector autoregressive models. A key prerequisite for VAR modelling was satisfied, as all variables were found to be integrated of order one. Moreover, the Johansen cointegration test confirmed the absence of long-term relationships among the variables in both models, thereby validating the chosen specification. Post-estimation diagnostics indicated that both models were reliable and stable, with all unit roots lying within the unit circle and residuals exhibiting no autocorrelation across tested lag orders.

The Granger causality tests yielded distinct insights for the two models. In model 1 (lead, EURO STOXX 50, WTI), all causal relationships were bidirectional, indicating that dependencies among d.ln.lead, d.ln.EUROSTOXX50, and d.ln.WTI operate in both directions. Consequently, each variable within model 1 provides valuable information for forecasting the others, thereby enhancing predictive accuracy and robustness.

In model 2 (zinc, EURO STOXX 50, WTI), d.ln.EUROSTOXX50 and d.ln.WTI were found to Granger-cause d.ln.zinc. Additionally, d.ln.WTI Granger-causes the lagged values of both d.ln.zinc and d.ln.EUROSTOXX50. All causal relationships in model 2 were bidirectional, with the sole exception of the link between d.ln.zinc and d.ln.EUROSTOXX50, which was unidirectional.

IRFs analysis revealed that shocks to individual variables tend to dissipate over time in both models. In model 1, the impact of a shock to d.ln.WTI on future values of d.ln.EUROSTOXX50 was greater than the reverse effect. Following a WTI shock, d.ln.EUROSTOXX50 exhibited a negative response from the first to the third lag, turning positive in the fourth lag. A similar pattern emerged in model 2, where the effect of a WTI shock on d.ln.EUROSTOXX50 exceeded the inverse.

FEVD indicated that the dominant share of variation in d.ln.lead, d.ln.zinc, d.ln.EUROSTOXX50, and d.ln.WTI is attributable to shocks originating within the respective variables. Although the influence of other variables increases over time, it remains comparatively minor throughout the analysed period. Identifying these causal linkages is crucial, as it may support the formation of heterogeneous asset classes and, ultimately, the construction of well-diversified portfolios. Furthermore, these findings contribute to the development or refinement of investment strategies that incorporate the instruments examined in this study.

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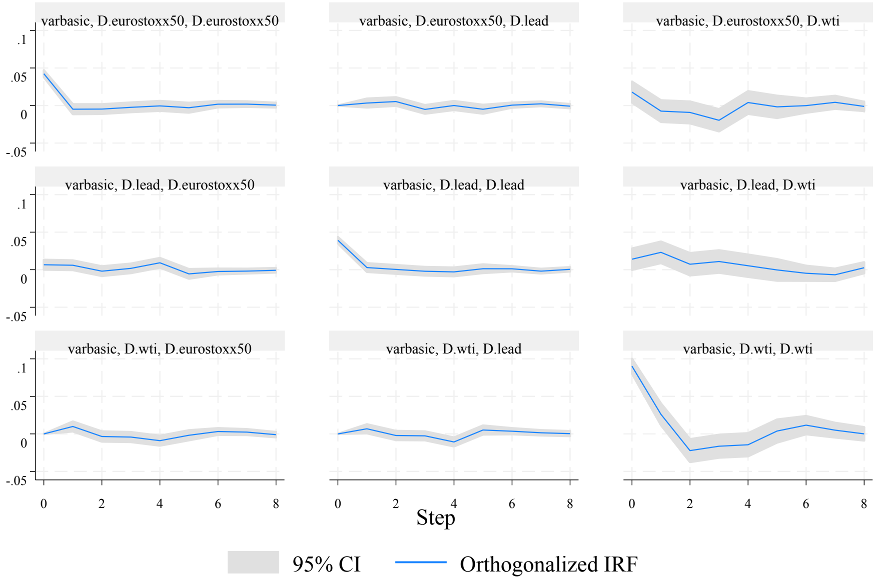
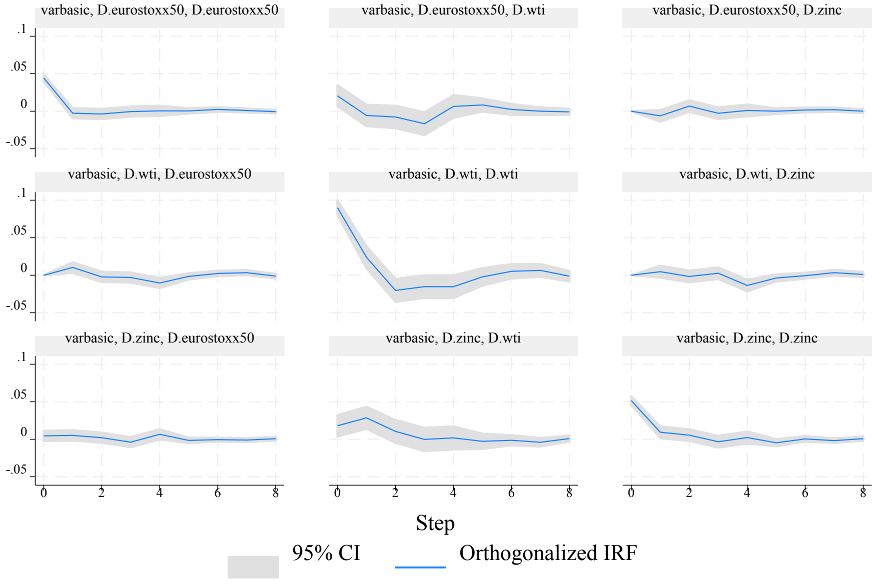
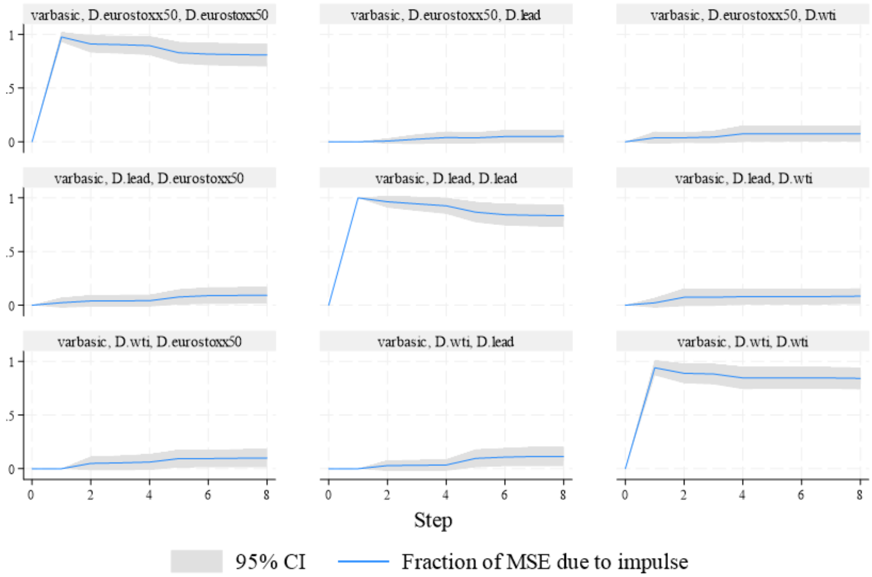
Appendix A



Appendix B



Appendix C



Appendix D